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How the Brain Gives Rise to Mathematics in Ontogeny and in Culture

Larry R. Vandervert

American Nonlinear Systems

Within the framework of Neurological Positivism (NP) this article describes how brain algorithms are translated into mathematics in ontogeny and in culture. The purpose is to address seemingly contradictory research findings that suggest that while mathematical axioms are innate, they are not the direct result of processes of selection. It is proposed that self-referencing feedback processes of maximum-power evolution guide the construction of algorithmic isomorphies between preadapted brain algorithms and mathematics. It is concluded that maximum-power evolution as described in NP offers mechanisms that make sense of findings that suggest that mathematical axioms are innate, yet not directly the result of selection as traditionally understood. It is concluded also that these mechanisms provide insight into the often intuitive nature of mathematical discovery.

Within the framework of Neurological Positivism (NP)[1] I have proposed that pure mathematics result from the exteriorization of the preadapted algorithmic organization of the brain (Vandervert, 1993, 1994). In NP the exteriorization of brain algorithms in the form of symbol systems (including those of the arts, religions, and sciences) occurs as a natural extension of maximum-power principle evolution (Lotka, 1945, Odum, 1988, Vandervert, 1991). Maximum-power evolution combines the processes of natural selection, natural opportunities of self-organization, and self-referencing (see, for example, Kauffman, 1993) toward the progressive accretion of advantage in

Requests for reprints should be sent to Larry R. Vandervert, American Nonlinear Systems, W. 711 Waverly Place, Spokane, Washington 99205–3271.

[1]In NP the algorithmic organization of the brain is the "positive" or basic data for what human beings can come to know. NP subsumes the traditional positivisms of Comte (social positivism), Mach (experiential positivism), and Schlick and Carnap (logical positivism). NP and the traditional positivisms are thought to be cultural-level isomorphic variants of the algorithmic organization of the preadapted brain.

optimal energy capturing and utilizing brain algorithms[2] in all organisms, and in human culture. In a nutshell, the progressive development of symbol systems arises as a continuance of the natural mechanisms of selection, self-organization, and self-referencing working at the cultural level.

In NP it is proposed that the brain algorithms that underlie the possibility for and exteriorization of mathematics are innate (Vandervert, 1993). This is not to say that mathematics as it appears in the development of individuals and in cultural form is innate. Rather, the relationship between brain algorithms, and the mathematics to which they give rise is one of algorithmic isomorphism (see Appendix)[Vandervert, 1991, 1992, 1993]. The creation of such algorithmic isomorphies is accomplished through the mechanisms of maximum-power evolution working at the cultural level as described above. I will describe the idea of algorithmic isomorphism in detail below.

Others (Giaquinto, 1992; Wynn and Bloom, 1992), following an entirely different strategy, have independently concluded that simple, initial mathematical knowledge appearing in animals and human infants is innate. Wynn and Bloom (1992) have dubbed such knowledge "psychological axioms." They have proposed that "the arithmetical and geometrical psychological axioms, although innate, are not themselves the direct object of natural selection; instead they are by-products of more general adaptations [perceptual–cognitive capacities]" (p. 414). The notion that certain brain processes related to mathematics are innate, but at the same time not due to natural selection (or other evolutionary processes), may seem patently self-contradictory. Wynn and Bloom explain this state of affairs by suggesting that the psychological axioms are inherent in their respective evolved perceptual–cognitive capacities, and that the "neural retooling" required for their expression "might have been quite minimal" (p. 412). However, Wynn and Bloom provide no mechanism for their neural retooling. One is left wondering what, precisely, is it that guides this hypothesized minimal neural retooling, and why does it occur at all?

Algorithmic Isomorphy

The purpose of this paper is to address these important questions by describing how NP's maximum-power principle mechanisms of algorithmic isomorphy account for cultural-level mathematical expression that is based

[2]Algorithms are methods of energy/information transfer that will provide solutions to problems; the first problem of evolution being the selection of algorithms that bestow advantage. Through evolutionary processes brain algorithms have come to guide energy flows that govern all perception, cognition, and behavior. Absolutely no brain processes occur without algorithmically guided energy flows. As Nobel laureate Dennis Gabor put it: "You cannot get something for nothing, not even an observation" (Tribus and McIrvine, 1971, p. 184).

upon innate brain algorithms. The concept of algorithmic isomorphy is central in NP and is equally central to the purpose of this paper, therefore I will digress for a moment to give it clear statement.

Within the systems–theoretical framework, Ludwig von Bertalanffy, the father of the systems movement, described algorithmic isomorphism among brains, machines, and mind:

> Obviously, logical operation performed in consciousness and the structure and function of the brain "is" not an electronic computer with transistors, wires, currents, programs and the rest. But in their formal structure they are comparable. Similar algorithms obtain: a computer (and a brain in its rational aspects) is, as it were, a materialization of logical operations, and vice versa logical operations are the conceptual counterpart of the functioning of a suitably constructed computer. This correspondence is a rather deep one. Boolean algebra and binary notation used in modern computers, the functioning of synapses according to the all-or-none law, and Aristotelian logic in thinking are structurally the same; the same algorithm or abstract model applies. (1967, p. 100)

Similarly, in NP mathematics "is" not neuron circuities, dendritic microprocesses, etc, but mathematics (the science of patterns, see Steen, 1988) is an externalized conceptual counterpart of the innate processing of pattern in the brain; the same algorithms apply.

The Evolutionary Mechanisms of Algorithmic Isomorphy

In NP the isomorphic relationship between the evolved brain algorithms that underpin science, mathematics, the arts, religion, etc on the one hand, and their respective symbol systems as they appear in cultural form on the other hand, is totally understandable as an outcome of *maximum-power principle evolution*. Within the framework of NP maximum-power principle evolution provides the mechanisms of algorithmic isomorphy (all isomorphy) between the innate brain algorithms of perception, cognition, etc., and mathematics. Thus in NP mathematics is not a mysterious "by-product" of innate brain algorithms, but a natural evolutionary outcome that is based upon them.

The maximum-power principle may be stated as follows: *Those systems that survive in the competition among alternative choices are those that develop more power inflow and use it to meet the needs of survival.* They do this by: (1) developing storages of high-quality energy; (2) feeding forward and feeding back work from the storages to increase inflows (self-referentially); (3) recycling materials as needed; (4) organizing control mechanisms that keep the system adapted and stable; (5) setting up exchanges with other systems to supply special energy needs; and (6) contributing useful work to the surrounding environmental system that helps maintain favorable conditions (Odum and

Odum, 1981, pp. 32–33). Lotka (1922, 1945) formulated the maximum-power principle, suggesting that systems prevail that develop designs that maximize the flow of useful energy through both positive and negative feedback. These feedback designs are sometimes called *autocatalytic* (self-releasing, feeding upon self, and self-organizing — Darwin's central thesis, form from preexisting form) [Odum, 1983, p. 6]. Thus in this evolutionary fashion the maximum-power principle constitutes the mechanism behind the algorithmic isomorphy *across* species. Now, we can extend this principle to the algorithmic isomorphy *across* brains, machines, and cultural-level notational systems like mathematics.

How Do Mathematics Arise From Innate Algorithms of the Brain?

The maximum-power principle is *itself* a macro principle of selection, self-referencing, and self-organization (Odum, 1988; Vandervert, 1991) that became encapsulated in the algorithmic organization of the human brain through evolutionary processes, thus giving rise to the mechanism and later possibility of the evolution of cultural-level algorithmic isomorphies in the neural subcircuitry of the brain. It is manifest in preadapted form, for example, in the self-organization of the perceptual constancies, cognitive systems (for example, Stadler and Kruse, 1990), and then in the continued autocatalytic emergence of cultural-level mental models (Vandervert 1991, 1993). It is my belief that the maximum-power exteriorization of brain algorithms in the form of symbols systems is associated with the emergence of the uniquely human neofrontocerebellar circuitry (MacLean, 1991).

The maximum-power autocatalytic relationship between evolved brain algorithms and the "retooled" subcircuitries that underlie symbol systems like mathematics is one of both *positive and negative feedback*, whereby energy capture and utilization is both progressively accelerated and stable (see especially features 1, 2, and 6 above). At the micro level it is suggested that the evolution of "retooled" computational patterns in ontogeny come about through neuron group selection as described by Edelman (1987).

Recognition of the fundamental mechanisms and characteristics of maximum-power-principle hierarchies leads to the central principle of NP governing the evolution of cultural-level mental models: *in its production of cultural-level mental-model brain circuitry designs, the algorithmic organization of the preadapted brain can increase energy inflows to itself most efficiently (maximum-power) by creating (feedforward) and feeding back those designs that are* ***similar*** *to itself*. Thus the brain is constrained to create its cultural-level symbolic models, like mathematics, in its own dynamical image, so to speak (again, form from preexisting form).

In NP these brain algorithmic based symbol systems represent the conceptual side of Wynn and Bloom's "retoolment," as it were, of brain subcircuitries that come into existence only because they in some way, given their overall cultural context, confer additional advantage. Therefore, the mechanisms of maximum-power evolution (especially self-referentiality here), (1) provide a way to understand how the algorithms that underlie mathematics can be innate, yet (2) appear through selection in a cultural form that is applicable to the same real world that we know through movement, perception, and cognition. We can now begin to understand why and how neural retooling in ontogeny and in culture would occur, and why mathematics would thereby not only come into existence, but continue to evolve. At the same time, the maximum-power principle applied to the evolution of culture as brain algorithm based accounts for the progressive acceleration of the development of culture. Not only can we humans "stand on the shoulders of giants"; each of us, through our capacity to think, stands on the shoulders of our own personal giant—our innate brain algorithms.

Conclusion and Discussion

In NP, mathematics is the result of both selection and self-referencing driven by the innate algorithmic organization of the brain. This explanation does not contradict Wynn and Bloom. On the contrary it supports their theoretical position by providing a mechanism through which mathematics can be understood in an innate but isomorphic form.

Further, by recognizing the isomorphic relationship between innate brain algorithms, and the symbol systems of culture, we can begin to address many perplexing questions. The history of mathematical and scientific discovery is punctuated if not best described by the role of intuition has played (see, for example, Einstein, 1981; Hadamard, 1945; Holton, 1979). What is intuition, and how does it work? How is it possible for an Einstein to arrive at scientific axioms *only* through *intuition* (his terms)? Why do mathematical discoveries often precede any conceivable application of them by perhaps a hundred years? How is it possible for a human being to simply sit *thinking* for a few hours, and thereby arrive at new mathematics?

If we assume that mathematics is somehow independent of our brains and its perceptual–cognitive capacities, these questions are probably unanswerable. However, if we recognize that mathematics has a natural evolutionary basis in the brain, answers can be formulated. In NP the algorithmic basis for all mathematics, scientific axioms, feats of engineering, and artist expression have been in existence in the brains of Homo sapiens for perhaps 50,000 years. It is only the perception of the right problem, along with the complementary motivation that will lead to the sustained thought necessary to exteriorize them in the appropriate emergent isomorphic forms. If this were not in some sense true, the great dream of eventually modeling consciousness and mind (these are not the same concepts in NP) in a mathematical form would be an impossibility. Finally, in NP, the intuitive leap that often leads to the discovery of scientific, and mathematical axioms is the result of the construction of fundamental isomorphies that connect preadapted brain algorithms with their cultural forms.

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Appendix

The meaning of isomorphy here is not the somewhat naive, geometrical isomorphy proposed, for example, by Wolfgang Köhler (Köhler and Held, 1949). Bertalanffy's example of algorithmic isomorphy is in keeping with *holonomic* brain theory that, "Understanding comes when the neurophysiologist searches for algorithms, such as computable [Fourier] transforms of sensory input" (Pribram, 1991, xxv–xxviii).

In NP's version of the maximum-power principle Troncale's (1988) systems-theoretical conception of isomorphy is adopted:

> There has been considerable evolution of the concept of systems level "isomorphy" since its first use in the field by the founders of research on a general theory of systems . . . some recent workers [for example, Troncale] have changed the word in its operational sense to a noun — "isomorphy" (pl. "isomorphies"). This change in part of speech draws attention to the possibility that these similarities [between different real systems] actually precede the origin of the system in many natural systems. Thus, the isomorphy may be so fundamental, and so essential to the emergence of any new systems from a multitude of parts over time that in order to achieve stability and long-term evolutionary potential many new systems-in-information "fall" into these isomorphic processes as obligatory optimal arrangement. Thus, in a sense, the isomorphies may be more "real" than the systems we normally term "real." Nature continually and recursively forms new systems at new scaler levels using the unique and immediate particulars in their novel combining capacities, but using the same tried-and-true structures and processes at the most general and abstract systems level. (p. 17)