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A Logico-mathematic, Structural Methodology: Part III, Theoretical, Evidential, and Corroborative Bases of a New Cognitive Unconscious for Sub-literal ($S_{ub}L_{it}$) Cognition and Language

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This second companion paper to a logico-mathematic, structural methodology (Haskell, 2003a) and its findings address theoretical issues underlying sub-literal ($S_{ub}L_{it}$) phenomena. The concept of a "cognitive psycho-dynamics" is introduced. In addition, research on masked priming and automatic activation of "chronic goals and motives" schemata are presented as initial and partial explanatory theoretical bases. Corroborating findings from fMRI and other neurological research suggest that some of the cognitive operations are biologically based. A biological evolutionary framework is then presented to explain the origin and development of $S_{ub}L_{it}$ cognition and lexical referents. Implications are discussed throughout.

Keywords: methodology, unconscious cognition, priming

In the first companion paper (Haskell, 2003b, this journal) to a logico-mathematic, structural methodology (see Haskell, 2003a, this journal), experimental design issues were addressed. This second paper presents theoretical issues and evidential bases that underlie unconscious or sub-literal ($S_{ub}L_{it}$) linguistic referents (see also Haskell, 1989, 1999a). The concept of cognitive psycho-dynamics is introduced as it applies to research on masked priming as well as to the literature on automatic activation of "chronic goals and motives" schemata.[1] These areas are intended as initial theoretical bases. In

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[1]The hyphenated form of the concept of a cognitive psycho-dynamics will be used when referring to sub-literal phenomena, thus separating it from therapeutic theories and placing it within a cognitive framework.

addition, corroborating findings from fMRI and other neurological research are presented for two of the more seemingly anomalous logico-mathematic, structural operations, suggesting the operations are biologically based.[2] A biological evolutionary framework is then presented to explain the origin and development of $S_{ub}L_{it}$ cognition and selection of lexical referents.

The Question of Unconscious Thought, Affect, and Reason

Unconscious Cognition Revisited

One primary question underlying sub-literal communication is: If motivated unconscious processing occurs, how cognitively sophisticated is it? At least since the 1950s, with the classic work of Dollard and Miller (1950), ongoing experimental studies on dozens of hypothesized psychodynamic functions have been conducted, but not without considerable controversy regarding their veridicality. In recent years, the controversy over motivated unconscious functions has engendered a number of books (e.g., Bowers and Meichenbaum, 1984; Dixon, 1981; Erdelyi, 1985; Hilgard, 1977; Lewicki, 1986; Shevrin et al. 1996), reviews (e.g., Erdelyi, 1984; Greenwald, 1992; Kihlstrom, 1984; Shevrin and Dickman, 1980) and special issues of journals devoted to the controversy (e.g., *American Psychologist, 47*, 1992; *Journal of Personality, 62*, 1994). Piaget (1973a) was perhaps the first to explicitly conceptualize an affective as well as a cognitive unconscious. While cognitive research on unconscious processing is now routine, most of it is not "psycho-dynamic friendly."

There continues to be wide disagreement about how sophisticated are analytically unconscious processes?[3] The answer to the above question, according to some (i.e., Greenwald, 1992), is that unconscious processes are not very "smart," being able to semantically process only a single word or two. How analytically complex are unconscious processes is a controversy that will not go away, with those on each side holding to their research findings. The varying definitions of "unconscious" notwithstanding, the opposing positions are largely based on differing methodologies, data, content variables, and degree of methodological rigor. Though it is not appropriate to review these positions here in detail, a few issues must briefly be addressed as they directly bear on $S_{ub}L_{it}$ reference.

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[2]For ease of exposition, henceforth "logico-mathematic, structural" will be shortened to logico-mathematic.

[3]While all analytically sophisticated processes are — by definition — complex, not all complex unconscious processes are analytically sophisticated. Reber's (1993) research on the unconscious learning of complex artificial grammars is an example of the former.

One issue concerns why findings on cognitive unconscious processing are so diametrically varied? From their own experimental orientation, Baars, Cohen, Bowers, and Berry (1992) and others (Sellen, and Norman, 1992; Stemberger, 1992) suggest that current laboratory conditions may not be adequate for replicating the underlying psychological dynamics of slips-of-the-tongue that spontaneously occur in everyday situations (see also Haskell, 2003c). Laboratory models of unconscious processing are often not sensitive to important encoding conditions.

In critiques of the research, there are three significant variables that spontaneously occur in natural settings that are typically not included in experimental designs. The first is determining whether a stimulus is adequately processed in terms of the length of time subjects are exposed to a task or stimulus (Merikle, 1998; Seamon, Marsh, and Brody, 1984). It is well established (e.g., Singley and Anderson, 1989) that the nature of the encoding process strongly influences how information can be stored, and how information is stored determines how it is subsequently retrieved.

A second issue concerns whether stimuli are meaningful to a subject (Haskell, 1986a, 1986b, 2003c; Merikle, 1998). It has been known for some time from dichotic listening experiments (Kimura, 1967) that *meaningful* content presented to the unattended ear will break through into consciousness or attentional focus. Similarly, a review by Johnston and Dark (1986) on selective attention has shown that a semantic analysis of words presented in a secondary channel only occurs if the words have significant relevance to the subject. Groeger (1988) has argued that meaningful material is encoded quite differently from non-meaningful material. For example, Merikle (1998) concludes on the basis of his review of the research that,

> the results of these experiments suggest that the way a stimulus is encoded varies depending on whether it is unconsciously or consciously perceived. For example, when a stimulus is unconsciously perceived, meaning or semantics is the predominant code. However, it seems that when a stimulus is consciously perceived, structural or surface characteristics become more important. (p. 10)

Personal meaning, then, is a significant variable that needs to be built-in to experiments on unconscious processing.

A third variable is knowledge base: a highly significant variable that influences what becomes perceived as meaningful for an individual. The knowledge base of individuals, however, is generally considered a variable that is too problematic to control in experiments.

The affective quality of stimuli is a fourth variable; it influences whether stimuli are sufficiently encoded in order to unconsciously influence memory and cognitive operations in ways that are "smart." Only slowly has the role of affect come to be recognized as integral to cognition (Abelson, 1963; Lazarus, 1982; Murphy and Zajonc, 1993; Piaget, 1973; Osgood, May, and Miron, 1975; Zajonc, 1980; also see implications section below).

Research conducted on everyday phenomena seems to show unconscious processes that involve more sophistication than those demonstrated in laboratory experiments. Drawing upon research from natural settings on phenomena like dissociation, implicit memory, amnesia, perceptual defense, recovery of memories from surgical anesthesia, hypnosis, etc., Bowers and Meichenbaum (1984), Kihlstrom (1987), Kihlstrom, Mulvaney, Tobias, and Tobias (2000), Kihlstrom, Bamhardt, and Tataryn (1992), Merikle and Daneman (1996) and others conclude that unconscious processing can be cognitively sophisticated. Kihlstrom et al. (1992) suggest,

> these unconscious influences may be quite different from those observed in subliminal perception (to take one example), for the simple reason that the events in question, although inaccessible to phenomenal awareness, may nonetheless be subject to quite complex cognitive processing. The sheer diversity of the available evidence for unconscious perception, memory, thought, and learning, is important precisely because an appreciation of *the full span of the psychological unconscious may provide an additional perspective on the matter of the analytic power of unconscious processes*. (p. 789, italics added)

Unfortunately, findings on unconscious psycho-dynamics tend to be viewed suspiciously. Accordingly, such findings evoke considerable reactions from the more traditional cognitive sciences strictly concerned with laboratory designs (see Haskell, 2003c). Clearly, findings derived from the logico-mathematic methodology (Haskell, 2003a) suggest a sophisticated set of unconscious linguistic and cognitive operations generated by meaningful social and psychological conditions. The results of the research just presented (and to be presented below) can be seen as support for $S_{ub}L_{it}$ findings and constitutes a new cognitive unconscious (Haskell, 2001c).[4]

The Question of the Default Level of Cognitive Processing

At the beginning of this paper, the issue of how "smart" is unconscious processing was briefly addressed, as was some of the methodological issues involved. Increasingly, some cognitive scientists suggest that most — or perhaps

[4]As this article was about to go to press, Hassin, Uleman, and Bargh (2005) edited *The New Unconscious* (New York: Oxford University Press, initially entitled *Unintended Thought 2: The New Unconscious*). The book reviews and extends the research literature on automatic v. controlled activation processes. Several of the chapters — especially chapter seven by John Kihlstrom, in a near reversal of previous conclusions — strongly suggest that unconscious processing is more sophisticated and volitional than is currently accepted by nearly all cognitive science. Indeed several of the authors argue that most thought is first constructed on an unconscious level (see sections below, *The Question of the Default Level of Cognitive Processing*; and *Automatic Activation of Chronic Goals and Motives: Masked Priming of Stereotypes*).

even all — thought is initially unconscious; that conscious thinking is merely the product of work done unconsciously. Indeed, as counterintuitive as it may seem, on the basis of reviewing the research, Reber (1993) maintains that

> *the unconscious should become the default condition.* That is, rather than putting the burden of demonstration on those who claim that a particular process was unconscious or that a given knowledge base was tacit, the burden of proof should go to those who argue that it was, in fact, conscious. In short, I will assume "the primacy of the implicit." We would do well to recognize that it is actually more surprising that any function is conscious than unconscious. Like many of the above proposals, this one is going to take a good bit of argument to be convincing. (p. 9, italics added)[5]

For analyses of the research and history upon which this view is predicated see Velmans (1991), Smith (2004), also Haskell (1999a, especially chapter eight).[6]

The implications of such a view — if only accepted in some significant measure — bear directly on sub-literal cognition and language. If much of thinking and affect is first processed unconsciously, then sub-literal cognition and language become primary in generating so-called literal language and thought.[7] It appears that somehow unconscious thought and affect remain attached to the literal narratives. Further, if most of thought is unconscious then this renders the concept of a cognitive psycho-dynamics almost transparent.

A Cognitive Psycho-dynamics

In analyzing sub-literal communication, two processes need to be clearly distinguished. The first is *how* unconscious cognition and emotion are generated. This entails the mechanics of the various cognitive operations presented in the logico-mathematic methodology (Haskell, 2003a). The second is *why* or for what purpose they are generated. The concept of a cognitive psycho-dynamics is employed for explaining underlying intentional processes that generate sub-literal referents and language. In effect, everyone employs psycho-dynamic explanations. For example, a statement of the class, "He or she did that because . . ." involves a cognitive psycho-dynamic explanation. All "because. . ." explanations

[5]Reber (personal communication) has backed off this view somewhat. In any case, it should be made clear that he does not accept $S_{ub}L_{it}$ cognition and language as a validly established phenomena.

[6]As an example, consider the analogy of consciousness being equivalent to one's computer screen. What is displayed there is equivalent to consciousness, but is not created there, of course; rather it is the product of the undergirding programs, with the latter being analogous to unconscious processing.

[7]For more on the distinction between literal v. figurative language that directly bares on $S_{ub}L_{it}$ analyses, see Haskell (2002a, 2005 in press).

about unconscious or inferred motivation that precipitate or influence conscious behaviors are psycho-dynamic ones. The concept of psycho-dynamic should be understood to refer to unconscious cognition and bears no relation to any therapeutic school or theory.[8]

As developed here, cognitive psycho-dynamics refers to thoughts and affective schemata that are inferred or hypothesized to be occurring on an unconscious (however defined) level that undergird the generation of $S_{ub}L_{it}$ reference. More specifically, cognitive psycho-dynamics can be seen as consistent with two areas of research on unconscious referents.

The first area is the experimental research of Bargh and Barndollar (1996) who suggest an automatic activation of chronic goals and motives to explain unconscious responses to stimuli. The second area concerns experimental work on speech errors or slips-of-the-tongue. Though the overwhelming body of research concludes there is no evidence that slips-of-the-tongue are generated from an unconsciously motivated base, the work of Baars, Cohen, Bower, and Berry (1992) leaves open the possibility based on what Baars has termed the competing plans hypothesis (see more below). The fundamental issue underlying the logico-mathematic method and sub-literal findings is whether unconsciously motivated and complex linguistic processing occurs.

Automatic Activation of Chronic Goals and Motives: Masked Priming of Stereotypes

Recent years have seen considerable development of experimental techniques for accessing unconscious material. One of these methods involves priming. Priming has stimulated an area of research on *automatic* versus *controlled* activation of unconscious stereotype schemata. Experimental work has also been conducted on the use of priming techniques (Draine and Greenwald, 1998) to elicit unconscious responses. Masked priming involves stimuli presented either outside of a subject's focal attention, or are presented so rapidly that stimuli

[8]To link sub-literal material to Freudian theory tends to relegate it to what one researcher (Kihlstrom, 1994) has correctly described as a "hermeneutic wonderland" (p. 684) of psychoanalytic interpretation. Having recently received a rejection of a manuscript I submitted to a well-known psychology journal on cognitive research relating to unconscious processing of phenomena which was subsequently published elsewhere (Haskell, 2003c), I commented on the work of a particular cognitive researcher, noting that his conclusions were a little more accepting of unconscious meanings and psycho-dynamics than those of his colleagues. As it turned out this researcher was one of the reviewers selected to critique my article. In his signed review — which is most unusual — he was emphatic in saying, "The fact that I adopt a wider range of unconscious processes . . . doesn't mean that I am a *closet Freudian*." I was taken aback at this comment because in describing his work in that paper I was extremely careful not to imply in any way that his work was related to Freudian ideas. Such reflex reactions to phenomena that are perceived to be associated with Freudian theory have served to retard research on unconscious reference for decades. Indeed, until recently in cognitive science the term "unconscious" was studiously avoided, using instead the terms non-conscious, implicit, and a host of others.

can not be consciously perceived but which, nevertheless, influence subsequent responses to the presenting stimuli.[9]

The research on automatic versus controlled activation of stereotype schemata indicates that stimuli presented either during an experiment — or present in everyday environments — can elicit automatic unconscious stereotypes. The research suggest two kinds of stereotypes, one cultural, the other individual or personal. It is generally accepted that automatically activated unconscious cultural stereotype schemata are created and conditioned by cultural transmission (including mass media, etc). Accordingly, automatically activated cultural schemata are thought to imply no volitional and individual motivation involved in their activation. In contrast, controlled activation of stereotype schemata is viewed as the consequence of individually held beliefs and implies some level of individual volition and motivation involved in activation.

Bargh and Barndollar (1996) suggest that under certain conditions unconscious schemata subserving goals and motives can also be automatically activated. Briefly, they view the unconscious as the repository of conditioned goals and motives that with frequent use or repetition become automated strategies (schemata) for responding to the environment. This automation is analogous to the automatic functioning of highly practiced motor skills. Accordingly, just as a "punch-drunk" professional fighter hears a bell and automatically begins to throw a punch, in a similar manner environmental stimuli can automatically activate chronic goal or motive schemata. Bargh terms this the "auto-motive" model and suggests that much of conscious behavior is the consequence of these automatically activated chronic goals and motives (see Bargh and Chartrand, 1999, for synopsis) that can be activated by environmental situations that function as priming stimuli.

Priming techniques in the automatic activation literature have been utilized to activate stereotype schemata on gender, social affiliation and achievement behaviors (e.g., Bargh and Gollwitzer, 1994). They have been more widely used, however, to automatically activate negative racial stereotypes (e.g., Devine, 1989). Wittenbrink, Judd, and Park (1997), for example, activated negative racial stereotypes in a group of volunteers by flashing either words or pictures that represented racial minorities onto a screen at a rate too brief for subjects to consciously "see" and report what appeared on the screen. With a second group, no images were flashed during presentation. The researchers induced anger in all subjects, and then asked both groups a series of questions. Subjects primed with images of an African American male were more likely

[9]There has been debate whether masked priming can activate cognitive processes without first being accessed through consciousness. Using event related potentials (ERPs) and functional magnetic resonance imaging (fMRI), Dehaene, Naccache, Le Clec'h, Koechlin, Mueller, Dehaene, van de Moortele, and Le Bihan (1998) and Naccache and Dehaene (2001) have demonstrated that unconsciously received stimuli can activate unconscious cognitive processing.

to respond in a hostile manner to annoying questions than were those also angered but who were not primed with racial images. Thus, racial bias was unconsciously (automatically) activated.

It has been clear for some time (see Haskell, 1986–87, 1999a) that the automatic activation experiments carried out with priming methods are laboratory parallels of $S_{ub}L_{it}$ findings and can be considered as explanatory models. Early on Haskell (1986–87) suggested that some of what appear to be racist comments (and/or slips-of-the-tongue) were automatic activations of racial schemata conditioned by the priming of negative cultural stereotypes in individuals. He noted that, "Listening to apparent casual verbal reports in a systematic and linguistically informed manner continues to reveal both personally current and psycho-socially *vestigial* remains of racial conditioning" (p. 75) . . . [which are] "precipitated and organized by conscious and non-conscious ideology, composed of negative stereotypes, attitudes and beliefs generated and sustained on a personal psychological and/or a sociocultural level" (p. 77). The problem of distinguishing personal from cultural schemata will be addressed below.

An illustration of priming an automatic activation of a racial stereotype occurred in a laboratory discussion group.[10] An older woman was the only African American and was perceived to be having more difficulty understanding and analyzing the group dynamics than the rest of the discussants. When this became known, and after a brief flurry of disjointed repartee, a topic was initiated by a White woman about */Dogs That Were Not Too Smart/* because they had been */Deprived/* as pups.[11] The specific breed of dog the woman referred to was */A Black Labrador Retriever/* she once had who was */Retarded/*.[12] Mapping the topic onto the narrative situation — along with the context of historical and extant negative stereotypes — suggests it was a

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[10]All references in brackets below, e.g., *[12.] Arithmetic Operations*, refer to sections in the Appendix of Haskell (2003a).

[11]In keeping with the formatting of the previous two articles in this series, textual material presented within slashes are verbatim and will be italicized and capitalized as would a heading.

[12]Methodologically, the fundamental questions that must be answered are: Why was the story about (a) a *black* dog, (b) the *effects of social deprivation*, (c) a *retarded dog*, and (d) the specific breed of dog, i.e., a *Labrador retriever*? In terms of a partial methodological validation, there are several significant linguistic linkages attached to this $S_{ub}L_{it}$ racial reference which suggest it was in fact a reference to the Black woman. First, the association to the color black is, of course, congruent with the Black woman being a priming stimulus. Second, the dog was said to be a slow learner. Third, it was a */Retriever/*. That it was a */Retriever/* involves semantic linkages to racial stereotype schemata. The term has historical semantic associative linkages to servant/slave behavior. That the dog was specifically a */Labrador/* retriever is likely a reference to the course being known as a laboratory course and thus a linguistic linkage between the literal topic and the narrative situation (*[3.4.] Transitional Linkage Operations; [8.4.] Paronymic Operations*). See next footnote for another paronymic operation.

$S_{ub}L_{it}$ reference (read: automatically activated racial schema) to the Black woman who was perceived as experiencing *difficulty learning*. It is well-known that a negative stereotype about many African Americans is that their childhood is one of social *deprivation*, which is believed by some to cause Blacks to have lower intelligence than Whites and thus more difficulties with learning.[13]

In priming and automatic activation terms, the fact that (a) the woman was African American, and (b) perceived as experiencing difficulty understanding group dynamics — were priming stimuli that automatically activated negative racial stereotype schemata in the White member who expressed the topic about /*A Black Labrador Retriever*/ who had difficulty learning. Analogically notated,

> early deprivation : the black Labrador retriever's later learning inability :: the early deprivation of the Black female's life : her perceived difficulty to learning in the group

An important unresolved issue in the literature is whether automaticity of unconscious motives and goals reflect a "fully" automatically activated cultural stereotype or whether it also represents *controlled* processes (see Kunde, Kiesel, and Hoffmann, 2003). Relative to the above experiment by Wittenbrink, Judd, and Park (1997), the question is whether those who were primed with racial images of the African American and who responded in a hostile manner to annoying questions exhibited an automatic or controlled response? In automatically activated schemata, individuals are not considered to hold "prejudicial" attitudes, whereas, conversely, controlled activation of individually held prejudicial attitudinal schemata are considered to reflect personal stereotypes. Bargh and Barndollar (1996), however, suggest the automatic verus controlled dichotomy is a false one, with the activation of stereotypes having both an automatic and a controlled or motivated component.[14]

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[13]Perhaps the most methodologically interesting is that the White female explained the dog was a /*Gorgeous*/ dog. This lexical item is important as it also indicates a linguistic paronymic linkage (*[8.4.] Paronymic Operations*) to an earlier session in which the Black female had given the White female an ashtray she had made for her. The White female accepted it explaining it was /*Gorgeous*/ (and thanked the Black woman — somewhat condescendingly — as one would thank a small child). It had also been stated by the Black female that she was originally from /*Georgia*/. This linguistic linkage is likely the $S_{ub}L_{it}$ equivalent of a paronym. Paronymic words derive from the same root or stem as another word, like *meanie* and *meaning*. Words that are graphically perceived to be paronymic-like are used to create sub-literal referents. Finally, it should be pointed out that many of these negative stereotypes that now may seem antiquated unfortunately are in fact still extant in some areas of society (see Haskell, 2001b).

[14]It is important both in terms of method and theory that it has been brought to my repeated attention that because a researcher recognizes these racial incidences it must reflect the activation of the researcher's own personal prejudice schemata; how else, the reasoning goes, could a person access, recognize and be attuned to so many stereotypes and racist perceptions

Applied to the $S_{ub}L_{it}$ racial narrative above, the question that remains is: Did the topic about a *black Labrador retriever* who had difficulty learning constitute an automatic activation of a racial *cultural* stereotype schema and thus not indicate individual prejudice, or did it reflect a controlled activation of an *individual* "belief" schema and thus indicated prejudice — or both?

While racism scales and questionnaires used to establish a kind of base line are an attempt to capture context variables and assess whether automatically activated racial material is essentially cultural or individually motivated, a more context-sensitive methodology is clearly needed.[15] For example, concerning the *black Labrador retriever* who had difficulty learning, the White woman was a self-professed "liberal" and was quite active in civil rights organizations, thus suggesting that she was not consciously prejudiced in the sense of being "racist," though she likely harbored some level of individual (or controlled) prejudicial racial stereotypes. This analysis would suggest that Bargh and Barndollar (1996) are perhaps correct in assuming that both automatic and controlled processes may operate simultaneously at least — and here is the rub — in some activated schemata. At this point, it seem reasonable to assume, as with slips-of-the-tongue, that some schemata are automatic, some controlled, and some a fusion of both automatic and controlled processing, with the latter two suggesting some magnitude of underlying racial psycho-dynamic.[16]

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and reasoning? This attribution is faulty. First, to maintain that recognizing sub-literal racial schemata reflects a researcher's personal prejudice is to say that all sub-literal analyses reflect a researcher's personal schemata. For example, to suggest that to recognize the repeated discussion of secret surveillance in a room (designed by the researcher!) with a one-way vision mirror reflects the researcher's affective concerns about being spied upon seems erroneous; likewise that narratives about *twins* and other references to 2s reflect the researcher's/trainer's own anxiety about being one of two researchers/trainers in a group (that the primary researcher/trainer initiated!) also seems contextually unwarranted. Second, it is the job of a social and cognitive psychologist to know and to recognize historical and current racial prejudices and stereotypes.

[15]I am compelled to add some explanatory background. First, growing up during times when racism was unfortunately socially acceptable, and being socialized into the full range of racial myths, images, and stereotypes, they remain cultural schemata in my memory. Second, a close friend and colleague for over thirty years who is African American has tutored me on subtle racial issues; this, along with sharing in negative racial experiences, has sensitized me to possible racial and racist behaviors, stereotypes, language, and other cues. Given the situated nature of knowledge, *in principle* there is always the possibility that my work may reflect — on some personal level — a racial "prejudice." In regard to this, in my article some eighteen years ago on revealing unconscious racial stereotypes in language (Haskell, 1986–87, p. 93), I noted: "I am . . . aware that it is perhaps because I am not "Black" that I may reach some of these conclusions . . . . The social psychological literature is replete with evidence demonstrating researcher bias" and influence. However, making such an attribution — just as with $S_{ub}L_{it}$ analyses in general — requires more than just an assertion; it requires a methodologically informed basis for the attribution.

[16]For more $S_{ub}L_{it}$ racial referents see Haskell (1986–87, 1999a, 2001a).

Experimental Research on Slips-of-the-tongue

There is a long standing corpus of research on what are generally called slips-of-the-tongue — more technically considered speech "errors" (Fromkin, 1973; Motley, 1985) — as well as research on action slips (Norman, 1981). Indeed, slips-of-the-tongue phenomena can be viewed as a kind of special case of sub-literal semantics. With nearly singular exceptions in both experimental psychology (e.g., Baars, Cohen, Bower, and Berry, 1992) and in linguistics (e.g., Jackendoff, 1999), there is little support for an unconscious volitional base for generating consciously unintended referents (as in so-called Freudian slips).[17] In the field of linguistics unintended referents are explained away as mechanical processing mistakes. However — as indicated above — Baars and others (Sellen and Norman, 1992; Stemberger, 1992) suggest that current laboratory conditions may not be adequate for replicating the psychological processes of slips that spontaneously occur in everyday situations. Baars et al. (1992) conclude that "From some of our findings . . . the most immediate conclusion might be that the Freudian hypothesis is just plain wrong. But that is too simple" (p. 308). Baars leaves the door open for new methods that may be less "blunt" (as he put it) than current ones.

Certainly, not all slips are unconsciously motivated; some are clearly mechanical mistakes or errors. Because many slips (likely even most) can be explained as mechanical or linguistic errors in executing the lexical, phonological, and syntactic systems, it does not logically follow — as the research literature suggests — that all slips are simply errors. This is a non sequitur of fundamental importance. Methodologically informed sub-literal findings point to a systemic class of linguistic productions with an unconscious intentionality subserved by a set of cognitive psycho-dynamics and operations. Further, it is likely that sub-literal language is subject to normal linguistic mechanisms that have been identified in speech error research, e.g., low-frequency words are "slipped" more often than are high-frequency words. There is no contradiction here. Contrary to the prevailing logic of experimental and computational research, there is no more contradiction in maintaining that sub-literal language exhibits volitional meaning and is carried out using lawful linguistic mechanisms than there is in maintaining that meaning expressed during everyday conversation is volitional and carried out unconsciously using lawful linguistic mechanisms. Except for certain research purposes, who would deny the latter?

To explain the psychodynamics underlying volitional or motivated slips-of-the-tongue, Baars et al. (1992) posit a competing plans hypothesis. The com-

[17]In a real sense, the consensus that "speech errors" are non motivated is artifactual to the design of the method used.

peting plans hypothesis assumes two intended but conflicting goals underlying some slips, one of which is unconscious, the other relatively conscious but not socially acceptable so it must be inhibited. According to this explanation, the consciously intended referent or goal fails and the unconscious and/or socially unacceptable goal (or meaning) overrides the initial plan and ends as either a consciously unintended referent as in a Freudian-slip or as a "garbled" speech error. The competing plans hypothesis, however, does not adequately address why there are two plans or — more importantly — why unconscious plans "motivationally" compete with one another or why and how one is inhibited while the other is not. As in most of cognitive science, competition in processing functions is explained in terms of external constraints, e.g., time pressures, and/or internal automatic processes and other non personal constraints.

Like the chronic-goals-and-motives conception of Bargh and Chartrand (1999), the competing plans hypothesis is essentially a psycho-dynamic explanation, only without Freudian dynamics imputed to it — though strictly speaking both explanations seem, in effect, to waffle on the issue. That is, unconscious motivational dynamics are replaced with more or less deterministic but undemonstrated processes. With the competing plans hypothesis, neither the existence nor the inhibition of the competing unconscious plans are adequately explained, e.g., how/why the competition was generated. Consequently, as in the automatic versus controlled activation research on stereotypes and chronic goals and motives, the issue of whether competing plans reflect "fully" automatically activated processes (without a personal basis) or whether they represent individually controlled processes (with a personal basis) remain unexplained.[18] Both sub-literal reference and its cognitive psycho-dynamics can be seen as consistent, but not specifically identified, with the mechanics claimed to underlie the competing plans hypothesis.

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[18]The issue of unconscious motivation seems to involve two fundamental conflicts within the research literatures. The first is maintaining a scientific — and therefore (seemingly necessarily) deterministic — approach while at the same time trying to give a nod to explanations that fit everyday experience that are volitional and non deterministic. Accepting "slips" as volitional and/or as non deterministic is seen as abandoning a scientific approach. The second conflict involves the belief that unconscious volition necessarily implies Freudian theory. There is an ironic twist to Freud's commitment to his approach that applies to cognitive science. While Freud observed "mental" operations in what I have referred to as his "cognitive trilogy" (see footnote 22), he was committed to demonstrating that unconscious processes were completely different from conscious ones, i.e., as dreaming is from waking cognition (Haskell, 1986a, 1986b). As Foulkes (1978) put it, "The very integrity of Freud's major discoveries is at stake . . . . [A]ny surrender, however small to rationalism, would ultimately serve to put the entire psychoanalytic enterprise in jeopardy" (p. 71). Similarly, cognitive science researchers assume they can not adopt a volitional explanation of slips for fear of putting their scientific approach in jeopardy. Thus, all of their theoretical maneuvers (i.e., competing plans hypothesis, the automatic activation of chronic goals and motives, methodological solipsism, etc.) serve to protect — as they understand it — a scientific approach to cognition.

Corroborating Research: Bio-neuro Origins of $S_{ub}L_{it}$ Numeric Operations

Some of the cognitive operations presented as part of the logico-mathematic methodology may seem to have no extra empirical bases.[19] Since a complete theory of $S_{ub}L_{it}$ phenomena is not yet developed — the next best option is to present findings from areas that can be seen as corroboration.[20] These will also function as background knowledge in assessing initial credibility of the logico-mathematic operations. Findings will be presented that support two logico-mathematic operations that seem to generate the most skepticism. The most controversial of the cognitive operations presented in Haskell (2003a) is that numbers contained in narratives have $S_{ub}L_{it}$ referents (see *[12.] Arithmetic Operations; [13.] Logico-mathematic Representation Operations*). The second is reversal operations where the reversing of names and initials express negation of the person sub-literally referenced (see *[10.] Reversal, Inversion, Opposition Operations*).

Animal and Human Infant Findings

Unconscious numeric cognition raises four fundamental empirical questions. The first is whether unconscious numeric mapping, tracking, and calculation in fact occur; second, is it possible for the brain to unconsciously "count" and "calculate"; third, is there neurobiological and evolutionary evidence that would support and subserve $S_{ub}L_{it}$ numeric findings; and fourth, why would these numeric phenomena occur? With regard to the first three questions, while calculation historically has been considered the province of conscious human beings, arithmetic abilities in nonhuman animals have been suggested for some time in rats and birds and in primates as well as (Koehler, 1951; Moyer and Landauer, 1967). It has also long been known that certain neurons have been hard-wired for specific functions (McCulloch and Pitts, 1943) with some responding not only to characteristics like angles (Hubel and Wiesel, 1959), but to specific numeric values (Thompson, Mayers, Robertson, and Patterson, 1970). Further, well-controlled research utilizing new experimental designs have identified a primitive arithmetic ability in nonhuman animals

[19]The theoretical base for this methodology has been in development for some time. Various parts of a theoretical base can be found in Haskell (1989, 2000, 2001a, 2001b, 2002a, 2002b). These works address a transformation of invariance function underlying not only $S_{ub}L_{it}$ operations but phenomena like analogical reasoning, similarity relations, etc.

[20]Unfortunately, corroborating $S_{ub}L_{it}$ findings from other domains generates other problems. Each domain of knowledge tends to create its own definition of methods and set of findings. These in turn constitute an internally structured epistemological framework that may be seen as "legitimately" addressed only from within the definitions and structure of that framework. Findings from other domains are often seen as somehow "not quite the same thing."

(Capaldi and Miller, 1988; Church and Meck, 1984). In addition, Woodruff and Premack's (1981) well-known work suggested not only a primitive understanding of whole numbers but also of fractions (proportionality) by chimpanzees. Within the last decade or so research has supported these findings (Boysen and Bernstson, 1996).

Carefully controlled for confounding variables, the pioneering and widely replicated experiments of Wynn (1992a) found that months-old infants have the ability to not only discriminate a number of objects but to add and subtract those objects. Wynn's findings at the time went against accepted wisdom concerning the cognitive development of children, which of course was largely dominated by Piagetian theory. Because both nonhuman animals and infants lack language and other cognitive skills thought to be necessary for "counting" and other mathematical calculations like addition and subtraction, this primitive numeric ability is technically referred to as *numerosity*. A comprehensive review of this area by Dehaene (1997) concluded that numerosity seems to be limited to "adding and subtracting" — and only up to three or four objects — after which numerosity takes the form of approximation. But the approximations are proportional and consistently close to actual magnitudes, which indicates a generalized calculation of numeric magnitude.

The work of Wynn and of Dehaene will serve as paradigmatic for illustrating unconscious numeric calculation. There seem to be specifically dedicated neurons and entire neurological circuits for various numeric operations like addition, subtraction, and approximation (Dehaene and Changeux, 1993; Stanescu–Cosson et al., 2000). It is thought that these abilities are likely hard-wired through evolutionary mechanisms (Wynn, 1998). Wynn's (1992a) research on addition and subtraction by four-and five-month-old infants was based on a simple but novel design that relied on the ability of infants to detect physically impossible events like two objects occupying the same space. Previous experiments by Baillargeon, Spelke, and Wasserman (1985) had shown that during the first months of life infants display strong puzzlement when they witness events that violate the fundamental laws of physics. For example, if infants see an object suspended in midair after losing its support, they spend increased time looking at the scene. Similarly, infants show surprise at a scene where two physical objects occupy the same space. Moreover — contrary to the Piagetian out-of-sight-out-of-mind claim — when an object is observed behind a screen, children show a strong reaction if it is not there when the screen is removed.

Wynn's (1998, 1992a) experiments involved infants watching a puppet theater where a hand appeared on one side of the stage holding a Mickey Mouse toy which the hand then placed on stage; subsequently a screen came up, hiding the toy. The hand again appeared with a second Mickey Mouse toy and deposited it behind the screen. This sequence of events was considered the concrete representation of the numeric operation of addition 1 + 1. The screen was removed, sometimes revealing the two toys, or sometimes only one toy (with the other removed through a hidden trap door behind the screen). Under the latter condition, infants look systematically longer at the impossible event of 1 + 1 = 1 than at the possible event, 1 + 1 = 2, which suggests that they were expecting to see two objects. That infants were carrying out computations, however, was not conclusive. To control for the possibility that infants were simply looking longer at a single object than at two identical ones, the 1 + 1 operations were reversed in a second group of infants who were presented with the physical operation of 2 – 1. The infants were now surprised to find two objects behind the screen (2 – 1 = 2); they looked at this situation up to three seconds longer than the possible event 2 – 1 = 1. Further, when Wynn ran a replication of the 1 + 1 experiment infants looked longer at the impossible outcome of three objects versus the possible outcome of two objects.

To further control for whether infants were simply building mental images, representations or models of scenes, and through memory comparing the scenes for changes, Dehaene and colleagues (Koechlin, Deheane, and Mehler, 1997) used a design similar to Wynn's except that objects were put on a slowly rotating turntable that kept the objects in motion even when they were hidden behind a screen. Because under this condition the infants could not form and retrieve a precise mental image of the scene, it made it impossible for them to predict where the objects would be when the screen was removed. Under these conditions, infants still found the impossible events 1 + 1= 1 and 2 – 1 = 2 surprising. The results showed that four-and-a-half-month-old infants are not confused by the objects being in motion, thus suggesting that infants do not depend on the precise location of objects for their numeric "counting." Apparently what infants are doing is constructing an abstract representation of two rotating objects with unpredictable locations; in short, they do not expect to find a precise representation of the objects. Thus infants know that 1 + 1 makes neither 1 nor 3, but precisely 2.

Not directly addressed by the researchers is that numeric calculation by months-old infants and nonhuman animals is almost by definition carried out unconsciously, i.e., automatically (unless one imputes a consciousness to nonhuman animals and months-old infants as somehow equivalent to adult's thought-out mathematical calculations). The question now is: Are there data that suggest unconscious processing of numbers by adults?

Neurological Findings

Going beyond behavioral measures, using event-related potentials (ERPs) and functional magnetic resonance imaging (fMRI) measures, along with a masked priming methodology, it has been demonstrated that consciously unperceived numeric stimuli not only activate visual circuits associated with numbers but that unconscious processing is being carried out as well (Dehaene, Naccache, Cohen, Le Bihan, Mangin, Poline, and Riviere, 2001; Dehaene, Naccache, Le Clec'h, Koechlin, Mueller, Dehaene, van de Moortele, and Le Bihan, 1998; Naccache and Dehaene, 2001). When presenting masked numbers prior to conscious attention at about 150 milliseconds, specialized visual areas recognize the shape of the numeric symbol but not its meaning; at about 190 milliseconds, however, numeric quantity is encoded and processed. This is indicated by the activation of other neurological circuits known to involve "calculation." It seems that unconscious processing of numbers is done in the ordinary course of everyday life. As Dehaene (1997) suggests, "Although we are not aware of all the automatic numeric computations that are continuously being handled in our brain circuits, their impact in our daily lives is certain and can be illustrated in numerous ways" (p. 79). If simple numeric calculations are found to have evolutionary precursors and specific neurological pathways, then it lends support for the veridicality of unconscious mapping, calculation, and approximations found in the verbal narratives presented in Haskell (1983, 2003a). In other words, unconscious numeric cognition is not just "psychological" and demonstrated only by behavioral data — it has a basis in corresponding neurological substrates.

One of the first indications of the neurological basis of number noted by Dehaene (1997) was the early and largely overlooked work of Thompson, Mayers, Robertson, and Patterson (1970): when cats were presented with a series of tones or light flashes particular cortical neurons fired only after the presentation of a certain *number* of events. One neuron reacted after six events of any kind — regardless of whether the events were six light flashes, six brief tones, or six longer tones. The sensory modality clearly was not relevant; the neuron only responded to a specific *number* of events (i.e., magnitude). Similarly, other neurons responded only to other specific numbers.

Moreover, Cohen, Dehaene, Chochon, Lehericy, and Naccache (2000) suggest that their research shows that two relatively separate numeric calculation systems exist, one for exact numeric representation and calculation, the other for approximate computations. Literal numeric narratives are often expressed in approximations to a reference that would not fit the intended $S_{ub}L_{it}$ referent if it were referenced exactly. For example, in a literal narrative, it may be said to be */About/* 10 or 11, or */Something Like That/*. The *about* and *something like that* make it possible to sub-literally reference simultaneously two subgroups of *10* discussants which would equal *11* total if the researcher/trainer is counted (see *[9.6.] Linguistic Tagging Operations*).

From his fMRI studies on both brain damaged and normal subjects, Dehaene suggests that exact calculation and multiplication is associated with left parietal area functions; comparison of numeric magnitude, however, is equally distributed across both hemispheres with a shift to the right hemisphere often seen. The right-hemisphere associated functioning seems incapable of calculation. When Arabic digits are presented in the right visual field, and therefore processed by the left hemisphere, subjects have no difficulty adding, subtracting, multiplying, and dividing. Such calculations are not possible when digits appear on the left visual field and are therefore processed by the right hemisphere. This calculation deficit persists even when subjects are asked to point toward an answer rather than to verbally respond. Finally, numeric approximation seems associated with right parietal area functioning (Cohen et al., 2000).

The numerosity research indicates that animals and infants track and monitor objects in their environment in order to arrive at a correct number. The research further indicates that tracking is not accomplished by object-identity but by the spatio-temporal trajectory of objects (Trick and Pylyshyn, 1994; Wynn, 1992b) that necessitates objects existing in, and moving from, distinct locations. Dehaene (1997) and Dehaene and Cohen (1991) point to occipito-parietal regions of the brain that rapidly extract the location of surrounding objects and encode the location regardless of the object's individual identity. It turns out that many of these spatial areas are the same areas activated by numeric stimuli, which suggests that much of mathematics may be derived from or founded on the perception of spatial information as has been generally theorized. Spatial location information about the objects is critical to setting up the representation of number and numeric reasoning (Dehaene, 1997). Significantly, numbers mentioned in narratives have been found to be consistently associated with a set of spatial coordinates (Haskell, 1987b; see also Haskell, 2003a, especially *[5.] Associational and Dimensional Operations*).

Certainly, tracking logically implies a dependence on spatial information. Dehaene , Posner, and Tucker (1994) review electrophysiological studies that suggest the existence of dedicated circuits for monitoring performance and which compensate for errors. Now, it is clear that tracking occurs in the unconscious numeric mappings and calculations in stories as indicated by the change in numbers in stories that correspond to changes in the conversational membership (see *[12]. Arithmetic Operations*; *[13].Logico-mathematic Representation Operations*). But more importantly, because the various linguistic and other associated characteristics of the numeric references in narratives are all internally consistent and integrally parallel to each other, there must be higher-order monitoring and tracking processes for this to occur.

Thus, it is crucial to emphasize that in order for a set of *consistent and structurally integral* numeric topics to be generated, each representation and its associated aspects and corresponding isomorphic meanings across the various transformations must somehow be cognitively (a) mapped, (b) tracked, and (c) stacked systemically throughout multiple levels of meaning and through the various story permutations, all remaining invariant with respect to the specific set of characteristics and meanings (see *[14.] Matrix and Lattice Structure Validation Operations; [15.] Multicorrelative Transformational Validation Operations*).

Reverals and Negation

In this section it will be suggested that cognitive reversal operations have a neurological and evolutionary basis which have evolved into higher-order processes. It will be recalled (Haskell, 2003a; see *[7.2.] Initials; [10.1.] Reversal Operations*) that reversals of names and initials seem to be a means of expressing negation. Expressing negation through reversal of actions is a form of communication based in evolution. For example, when motioning to a dog to go outdoors, the dog may *turn its head and look away* from the door. Similarly, it is well-known that prior to developing language, children often simply *turn their backs* to a person to whom they want to express "*no*." Having no language, this is a way of expressing "*no*" or negation via action. It is suggested that this early mode of communication is still cognitively operative despite humans having evolved a more sophisticated form of linguistic communication. Language, after all, is a relative new-comer in evolutionary terms and likely retains a continuity with earlier modes of expression.

If the various forms of expressing negation are evolutionarily based, then there should be neurological evidence of reversals. One possible manifestation of this is the common observation that up until about eight years of age children often reverse letters of the alphabet like *b* and *d* and *p* and *q*. Past about eight years old, reversals are typically not observed except among dyslexic children and adults.

Cognitive reversals are more ubiquitous than might be thought. For example, most people have some degree of writing ability with their non dominant hand. It has been known for some time that when right handed (or conversely, left handed) people write with their opposite hand, it often leads to reversing of letters, resulting in what is known as mirror image writing where a letter written normally as e becomes written in the reverse as ɘ. The well-known German name for this is *Spiegelschrift* (Harris, 1980).

Understanding normal cognitive functioning can often be enhanced by studying so-called abnormal or anomalous states. Though mirror writing can be done purposefully, under certain conditions it is involuntary (and is not the simple consequence of hand position). For example, people with injuries to their left hemisphere lose their ability to read rightward, but those who have an intact right hemisphere can readily develop the ability to read leftward (Heilman, Howell, Valenstein, and Rothi, 1980). Schott (1980) has described patients who, because of damage to their (dominant) right arm had to learn to write with their non dominant left hand. Most of these individuals, in addition to mirror writing, began to turn door knobs, jar lids, and screwdrivers in the reverse direction; most often they reversed the letters *d*, *b*, *s*, *z*, and *p* (just as young children often do, see below). And some of these patients reversed numbers as well.

An experiment described by Bradshaw, Bradley, and Patterson (1976) suggests just how common cognitive reversals are. Monkeys trained to recognize by touch the figure ⊂ with their left hand, when tested for transfer with their right hand preferred the reverse figure ⊃. There are other related findings on reversals as well. One of the few data from early studies of a split brain patient (Levy, 1977) showed that when the letters *K I D* were presented to the disconnected right hemisphere they were *reversed.* The potential importance of these finding is suggestive of a neural basis for the two operations from Haskell (2003a) where the researcher's/trainer's initials *R.H.* were reversed to *H.R.* by selecting the name of the novelist Harold Robbins (see *[7.2.] Initials*) as well as with the name of the journalist Harry Reasoner (see *[10.1.] Reversal Operations*). And recall from just above that in mirror writing letters are reversed in *orientation* as when **e** becomes written **ɘ**. Reversed order of letters is not often noted in the neurological literature. One direct neurological basis for reversing letter order comes from a the case of a patient who had encephalitis as a young man (Heilman et al., 1980). In a striking similarity to the reversing of letter order above, the patient not only exhibited mirror writing and reversal of *letter orientation*, but also reversal of the *letter order*, reversing *PH* to *HP*. Admittedly this is a singular piece of evidence, but given other instances of reversals, it is intriguing support for further research into the neurological basis of reversal operations for $S_{ub}L_{it}$ reference.[21]

Freud (1900/1954) too, observed reversals in dream content and described them as expressing negation. He observed that the word "no" does not appear linguistically in dreams. That is, a *negative* is expressed in non verbal images, not with language. He cites dreams where "no" is expressed in the dream by the *reversing* of things, and/or by the "turning *around the other way*"

[21]Reversals are sometimes consciously used symbolically, as well. Claude Levi–Strauss (1963) reported instances of inversion operations among preliterate peoples. For example, in Hawaii the death of a chief was marked by a ceremony where male participants inverted the wearing of their loin cloths from around their genitals to around their necks. This vestmentary *inversion* from a location low on the body to a reverse location high on the body signified and was accompanied by a change in sexual license. Levi–Strauss also observed that preliterate people in unconsciously constructing myths have a tendency to reverse things to express an *opposite* thought (see also Haskell, 1986b)

of objects as well as by turning things "*up side down*" (p. 326; see also Haskell, 1986b).[22]

While the above ideas point to a neurological machinery subserving reversals — as well as evolutionary origins that subserve numeric processing of cognitive reversals — they do not explain the *motivation* generating the reversals. Such evidence strongly suggests the mechanics of how the operations are possible — not the psychological why or the psycho-dynamics underlying the operations. Nevertheless, the findings provide some corroborative support and evidential plausibility for the veridicality of $S_{ub}L_{it}$ operations independent of their methodologically-based validity.

Corroborating Research: Mathematic and Algebraic Cognition

As noted in the introduction of this paper, while the logico-mathematic method and its $S_{ub}L_{it}$ numeric findings stand on their own, because of the anomalous claims and findings as well as the divergence from standard experimental and epistemological paradigms, more than typical instantiation is required. Some corroborating research and theoretical bases need to be provided. This section on mathematical findings will provide that instantiation. As briefly mentioned, there have been attempts to demonstrate a logico-mathematic base to cognition as well as to develop logico-mathematic methods. With the arguable exceptions of sensory psychology and psychophysics, most attempts have failed. In fact, the very search for a logico-mathematic basis of cognition seems to have been abandoned some time ago.[23] Yet, why should arithmetic and algebraic structures be reflected in cognition? It is one thing to posit that arithmetic and algebraic structures undergird the narratives that are inherent in cognition, it is another matter to *demonstrate* that they may be inherent. Indeed, it is increasingly suggested that logico-mathematic ability is in large measure an artifact of Western culture (see Gardner, 1993). Historically, mathematicians have weighed in on both sides of this issue.

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[22]Freud observed numerous unconscious linguistic operations in everyday life throughout what can be seen as his "cognitive trilogy" i.e., *Interpretation of Dreams* (1900/1954), *Jokes and Their Relation to the Unconscious* (1963), and *The Psychopathology of Everyday Life* (1914/1960). Despite his observations, he did not understand the cognitive implications of what he observed (Haskell, 1999a, 2001b). Freud viewed these "cognitive operations" as merely representations — that did not function cognitively. Moreover, he clearly stated that the many apparent "cognitive operations" mentioned in his book, *The Psychopathology of Everyday Life*, are "of an entirely popular character" and that the book "merely aims by the accumulation of examples, at paving the way for the necessary assumption of unconscious yet operative mental processes, and it avoids all theoretical considerations on the nature of this unconscious" (p. 272). Freud never did develop a theoretical basis for the linguistic operations described in his trilogy.

[23]Perhaps as testament to the lack of research interest in this area the journal of *Mathematical Cognition*, edited by Brian Butterworth and Lisa Cipolotti and published by Psychology Press, no longer exists.

Basic Mathematical Constructs

In an extensive exposition Lakoff and Núñez (2000) have advanced the understanding of the "inherent" cognitive bases subserving mathematics. While it is not claimed that all higher mathematical operations are cognitively inherent, e.g., the concept of zero, the concept of infinity, square roots, logarithms, infinite series, the concept of an empty set, complex numbers, transfinite numbers, etc., Lakoff and Núñez have shown that *fundamental* mathematical operations seem to be both cognitive and neurological in that they evolve from internal image schemata; and that the cognitive structure of mathematics "makes use of the kind of conceptual apparatus that is the stuff of ordinary everyday thought" (p. 30) and that much of this occurs unconsciously. They further argue that many cognitive operations that are not specifically mathematical form the basis of mathematical ideas, including "basic spatial relations, groupings, small quantities, motion, distributions of things in space, changes, bodily orientations, basic manipulations of objects (e.g., rotating and stretching), iterated actions, and so on" (p. 28); that research in cognitive linguistics shows that spatial relations decompose into conceptual primitives called image schemata; and further that these conceptual primitives seem to be universal.

More specifically, Lakoff and Núñez maintain that the mathematical concept of a class is based on the everyday concept of a collection of objects within a bounded region of space. For example, given two Container Schemas A and B and an object X, if A is in B and X is in A, then X is in B, and given two Container Schemas A and B and an object Y, if A is in B and Y is outside of B, then Y is outside of A. In other words,

> the formation of a collection or pile of objects requires conceptualizing that collection as a container — that is, a bounded region of space with an interior, an exterior, and a boundary — either physical or imagined. When we conceptualize numbers as collections, we project the logic of collections onto numbers. In this way, experiences like grouping that correlate with simple numbers give further logical structure to an expanded notion of number. (p. 54)

Moreover, they propose that the arithmetic operations of adding and subtracting are based on sensory-motor operations as when more objects are placed into a collection or objects are subtracted from a collection.[24] Along

[24]Compatible with the underlying processes of $S_{ub}L_{it}$ operations is the work of Johnson (1990), and Lakoff and Johnson (1999). Like Piaget before them, they posit a bodily sensory motor basis to thinking (see conclusion and implications sections below) along with an affective base underlying otherwise abstract schemata. From their cognitive perspective, Lakoff and Núñez assume that certain aspects of mathematics are "based on" sensory-motor operations. Some mathematicians, however, assume a different ontology. Anthony Badalamenti suggests "that such physical operations are among the first expressions of the ability or that such actions are linked to activating the ability in the first place" (personal communication, e-mail, April 30, 2003).

with explaining many other mathematical concepts in cognitive terms, Lakoff and Núñez also argue that the mathematical concept of recursion is based on the everyday concept of a repeated action.

Algebraic Structures

Pertinent to the demonstration by Haskell and Badalamenti (2003) of an algebraic structure undergirding the logico-mathematic methodology — is the explanation by Lakoff and Núñez of the cognitive significance of algebraic structures. First, they suggest that "the field of abstract algebra is a magnificent case study of the use of fundamental cognitive mechanisms in mathematical cognition" (p. 119). As the study of mathematical form or abstract structure, algebra does not include numbers, only "elements"; nor does it include specific arithmetic operations like addition and multiplication — rather it makes use only of "operations." In short, the abstract elements, operations, and "laws" of algebra are independent of arithmetic.[25]

Lakoff and Núñez further claim that what renders algebra a cardinal discipline is its relationship to other branches of mathematics. Algebraic structures are conceptualized as the abstract "essences" of other mathematical structures. To arrive at their findings, they developed a collection of what they call Algebraic Essence metaphors. These are conceptual mappings from algebraic structures to structures in other mathematical domains — and by generalization to non mathematical domains.[26] They contend that Algebraic Essence metaphors and their corresponding conceptual blends "are an essen-

[25]As a technical point, it can be argued that algebra does not include numbers, only "elements." According to Anthony Badalamenti "algebra does include numbers and arithmetic operations but it goes on to generalize the structures where such things as numbers and arithmetic occur; for example ordinary arithmetic becomes an instance of operations within a 'field' where the concept of number is replaced by 'element' and the arithmetic operations are replaced by abstract definitions that generalize the operations of arithmetic" (personal communication, e-mail, April 30, 2003). Further, that algebra's abstract elements, operations, and "laws" are independent of arithmetic, as a mathematician Badalamenti would say that they are "not independent of, but a generalization of arithmetic. The major thrust of math, especially in the 20th century, is to find underlying universal structures that generalize structures that arise from ordinary experience such as arithmetic, sets and the ideas of common sense logic."

[26]While Lakoff uses the term "metaphor," for readers not familiar with the literature the term was originally conceptualized as a simple linguistic trope or figure of speech. It is now widely recognized as reflecting a fundamental cognitive operation (Haskell, 1987a). Over and above this, the way Lakoff and Núñez apply the term, the concept of analogical reasoning could equally — and perhaps more appropriately — have been used (since they are describing mappings of structure). Until recently, the two literatures (metaphoric and analogical) have been relatively separate. The term "mapping" systematically originated in analogical reasoning research. Here "structure" is a core analytical and empirical concept. Both "metaphoric" and "analogical" reasoning are now accepted as fundamental to all thinking. The common link between the two is the concept of similarity. For some time Haskell (1989, 2002a) has

tial tool for seeing identical structures in very different mathematical domains and for formulating them precisely." As most mathematicians would agree, since structural mappings "preserve inferential structure, anything proved about the algebraic structure will also hold for the mathematical structure it is mapped onto" (p. 117).[27] Recall that it is this mapping of numeric references in literal narratives onto the actual composition of the narrative membership that results in revealing an algebraic structure, and suggests the validity of the $S_{ub}L_{it}$ referents.

Lakoff and Núñez all but explicitly recognize that algebraic operations form the basis for the abstract structure of thought. The problem, they suggest, is that "Cognitive science must explain how abstract reason is possible and how it is possible to have abstract concepts and to understand them. This is anything but a trivial enterprise" (p. 347). Similarly Verene (1981), a philosopher, has lamented, "It is a scandal of logical thought that it cannot make clear the very basis upon which judgment itself is possible" (p. 174). Indeed, it is an old problem in philosophy. Harald Hoffding (1905), a psychologist and philosopher, wrote extensively on this problem. He says,

> There are typical or general ideas, only in the sense that we can make a concrete individual idea serve as an example or representative of a whole series of individual ideas. The generality of an idea will, then, mean nothing more than its fitness to be employed as example or representative. But it still remains to be asked, *What is the psychological process by which an idea comes thus to be set up as representative?* (1905a, p.166, italics added)

The numeric data in Haskell (1982, 1983, 1984, 2003a) suggest that there are abstract structures (numeric schemata) into which various narrative contents are "inserted." How this is cognitively accomplished, as noted above, is not known (see Haskell, 2000, 2001a, for more). It follows that, given the algebraic structure found subserving the logico-mathematic method and its $S_{ub}L_{it}$ referents, future research on the method and its findings is central to cognitive science for both understanding cognitive structure and, more specifically, for understanding thinking and reasoning.

Also supportive is the work of Marcus (2001). Based on his analysis of connectionism and computational data and theory he suggests that the brain

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conceptualized the cognitive processes underlying metaphoric, analogical reasoning, and $S_{ub}L_{it}$ cognition to be an invariance transformation function. Thus much of the logico-mathematic method and $S_{ub}L_{it}$ phenomena are a more basic but more complex (dare it be said, "natural") form of analogical reasoning.

[27]"This is very important in mathematics because one solution to a problem at a high level of abstraction becomes a solution to its specializations where the algebraic form is "instantiated" and thus makes for immense economy of effort" (Anthony Badalamenti, personal communication, April 30, 2003). In terms of brain and cognitive function the creation of economy would have been evolutionarily significant.

inherently functions algebraically. Related research by Dehaene (1997) — who notes the work of Hittmair–Delazer, Semenza, and Denes (1994) as well as the work of of Hittmair–Delazer, Sailer, and Benke (1995) — finds that when people become acalculic they do not necessarily lose their knowledge of algebra: there may exist neuronal circuits responsible for algebraic functions that are largely independent of those involved in mental calculation (see more below). Such research, too, is supportive of the Haskell and Badalamenti (2003) findings that this series of $S_{ub}L_{it}$ numeric topics exhibits an algebraic structure.

Given all of the above, it would seem warranted to resume the search for the logico-mathematic structures extant in at least some aspects of cognition. As Haskell and Badalamenti (2003) suggest,

> There are many possible uses of generalizing the possible fact that cognition itself operates via the mathematical concepts that have been taken to be the end products of cognition rather than their progenitors. That is, the narrative that one uses to define the idea of a semigroup is itself formed by operations that presuppose the concept itself as a principle of operation . . . . If the present inference that some of mathematics is itself the organizing principle of cognition is correct, then a blueprint for deeper and more functional mind maps presents itself. This could mean, for example, that the present semigroup structure could be used as a template for replicating an aspect of human cognition. (p. 392)

Finally, for all their seemingly anomalous characteristics, perhaps the most interesting aspect of the series of numeric narratives is neither their sub-literal semantics, nor their methodological operations, nor their ungirding algebraic structure. As Haskell and Badalamenti (2003) observed, "While this alone is significant, it is more interesting to note that the series of stories . . . [was] generated by more than one individual" (p. 8). This means that the algebraic structure was the consequence of multiple minds. The authors conclude that this only "makes sense if it is assumed that this algebraic structure emanates from neurologically based substrates shared by all humans" (p. 8). The research on processing mathematic operations demonstrates just such shared neurological substrates.

Concluding Comments

Certainly the evolutionary precursors to sub-literal numeric cognition are not identical to their evolved higher-order incarnations. The numeric findings in the verbal narratives presented in Haskell (2003a) are more sophisticated than their presumably hard-wired origins. The question is: How might that increased sophistication be explained? First, unlike the findings with nonhuman animals and infants, numeric references were generated by adults who had been exposed to (a) learning at least some mathematical operations; they (b) had more experience in encoding spatio-temporal situations; and as a consequence (c) have developed a more integral set of related neurological circuitry. Second, and most importantly, $S_{ub}L_{it}$ numeric references are more sophisticated than their precursors because both unconscious and conscious cognitive processes are very likely simultaneously involved in generating $S_{ub}L_{it}$ phenomena. As Bargh and Barndollar (1996) have insightfully observed, a wide-spread assumption in the research on unconscious processing is that conscious and unconscious processes are distinct, or at least that lower- level processes exert their influence serially, not simultaneously.

Considerable research (including $S_{ub}L_{it}$ findings) suggests that both conscious and unconscious processes influence each other with massively reciprocal reentry feedback loops. This latter model is more congruent with the evolutionary biological view of brain functioning put forth by Edelman (1992), as well as with what Kosslyn and Koenig (1995) have termed a "wet brain" approach to cognition, as opposed to the "dry" conventional computational models.[28] Thus, sophisticated numeric mapping, tracking, and calculation found in narratives make sense if they are generated by multiple neurological circuits operating simultaneously on both conscious and unconscious levels. Other models for this include the multiple levels of unconscious circuitry leading to the conscious experience of vision laid out by Marr (1983). Another model is Triesman's (1985) preattentive processing.

A well-known and rather dramatic illustration of unconscious processing from vision research concerns blindsight (Weiskrantz, 1995), where patients with lesions in the cortical visual areas report being unable to see an object presented to them. Yet when asked to "guess" what the object was, subjects are able to do so at better than chance levels. Similar findings have been reported with nonhuman animals. In other words, on "some unconscious level" the information was neurologically encoded and processed. Thus, while the conscious recognition of objects is not available, unconscious knowledge of objects is nevertheless being processed. The processes that undergird "guessing" are most likely the cognitive equivalent of the processes undergirding the unconscious processing of the selection and mapping of literal numbers in narratives.

How unconscious numeric references work can be understood within Dehaene's (1992) triple-code theory in which human numeric cognition is viewed as a layered set of modular architectures with the more primitive preverbal representations of approximate numeric magnitudes subserving the progressive emergence of a language and numeric notation/symbolic calcula-

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[28]Edelman describes a kind of hyper-feedback loop among groups of neurons that makes possible coordination and integration among what would otherwise be disconnected percepts. He defines reentry as a dynamic ongoing process of recursive signaling across massively parallel reciprocal groups of neurological circuits.

tion-dependent abilities by an algorithmic transcoding of one into the other. Dehaene notes that "Although the current research trend is to explore the deeper layers, or the 'primitives' of numeric cognition" (p. 35), future work will be on higher order mathematical abilities. The unconscious processing of numbers in the logico-mathematic method probably functions on a similar transcoding, progressive and layered basis.

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Finally, what purpose does unconscious numeric mapping and calculation as expressed in narratives serve? It is well established that nonhuman animals possess the ability for tracking, but findings that months-old infants have an innate ability to track objects is fairly recent (see Chiang and Wynn, 2000). From an evolutionary framework, then, the answer to the question is that just as numeric mapping, calculation, and approximation of objects (including predators) in an animal's environment function as *tracking mechanisms*, so too is it likely that unconscious mapping, calculation, and approximations by humans during conversation serve the function of *unconscious tracking* of total group membership including various alliances or coalitions that are or may be operating.

Certainly the development of unconscious cognitive skills for tracking would have had important survival benefits for early humans (see Haskell, 2002b). Various researchers (Cords, 1997; Dunbar, 1996) have argued about the importance of animal and early human tracking of group membership in order to assess alliances, recognize disputes, note absences, etc. Just as this evolutionary analysis would predict, virtually all unconscious mapping and calculating found in the stories and topics were about tracking group membership both in terms of the total number of members as well as for dominance relations, gender, sexual tensions, age differentials, as well as for exchange of favors — all equally important variables for both human and nonhuman animals to function in a social environment of increasing complexity. It has been known for some time that nonhuman animals from vampire bats (Wilkinson, 1984) to primates (Seyfarth and Cheney, 1984) engage in reciprocal favor-giving. To do so requires tracking of social dynamics (see Buss, 1999).

But why would numeric tracking be carried out unconsciously rather than consciously? One answer to this question is that basic numeric tracking skills evolved among nonhuman animals before the advent of modern homo sapiens, and thus laid down neurological circuits prior to "conscious" thought and language. A second more serious question is: Why are numeric tracking and communication about feelings and concerns still carried out unconsciously now that modern human consciousness and language have evolved?[29] The answer to this question may involve the strategic deception of others, perhaps analogous to nonhuman animals evolving camouflage and other deceptive practices to adapt to the interpersonal realities of social living.

Based on Trivers's (1971) well-known research on reciprocal altruism showing that nonhuman animals must devise strategies for detecting "cheating" on payments due for past favors, human animals must also devise strategies for hiding their cheating. The argument is that with humans, consciously trying to conceal cheating is not all that effective. Subtle cues are invariably leaked to others. In other words — and contrary to what is generally thought — humans are not all that effective at lying. To resolve this problem, humans must deceive themselves so that they believe what they are saying. Accordingly, self deception allows for more effective social deception.

Implications

In addition to the primary empirical and theoretical issues discussed in this paper, the logico-mathematic methodology has a number of implications, only a few of which can be highlighted here, the most fundamental of which is the concept of similarity relations (Goodman, 1952; Haskell, 2005 in press; Osgood, 1949; Medin, Goldstone, and Gentner, 1990; Shanon, 1988; Shepard, 1987, 1994; Vosniadou and Ortony, 1989; Wallach, 1958) in cognition and reasoning. Similarity relations, analogical/metaphorical reasoning and transfer functions are all integral to sub-literal reference (see Haskell, 2001a). Nearly equal in importance is the burgeoning research on analogical reasoning (Gick and Holyoak, 1983; Rumelhart and Norman, 1981; Sternberg, 1977), structure mapping (Gentner, 1983), and metaphorical comprehension and reasoning (Haskell, 1987a; Honeck and Hoffman, 1980; Lakoff and Johnson, 1980), isomorphic relations (Hayes and Simon, 1977), as well as transfer of invariance operations (Haskell, 1989, 2000, 2004).

Further, the methodology bears directly on research into abstract story grammar and cognitive schemata (Thorndyke, 1977; Thorndyke and Hayes–Roth, 1979; Winograd, 1977), and the relation of narrative to language (Turner, 1997, 2001). Sub-literal narrative analysis is also directly relevant to the field of cognitive linguistics (e.g., Jackendoff, 1999, 2002; Langacker, 1987), and to the research on phonetic symbolism (Brown, 1958; French, 1977), as well as to aspects of language and literary theory (Culler, 1988; Haskell, 2003a; see

[29] I which to thank my colleague David L. Smith for leading me to the biological evolution literature for the possible origins of $S_{ub}L_{it}$ phenomena. He is currently in the process of developing an evolutionary framework for $S_{ub}L_{it}$ communication (see Smith, 2004).

[8.1.] Homophonic Operations; [8.1.2.] Oronymic Operations; [8.1.3.] Phonetic Operations; [8.4.] Paronymic Operations).

Of major theoretical interest is the role of emotion in cognition. The study of affect in cognitive science has had a long and controversial history (see Abelson, 1963; Lazarus, 1982; Murphy and Zajonc, 1993; Osgood, May, and Miron, 1975; Zajonc, 1980). Piaget (1973b) believed that a significant concern of the social and behavioral sciences is "trying to characterize affective life in relation to cognitive functions [insofar as they relate to structure] and especially of defining their interrelation in the actual functioning of behavior" (p. 39). More recently, emotion is recognized as playing a significant role in cognitive processes (e.g., Damasio, 1995; Kihlstrom et al. 2000; Kitayama, 1990; Kostandov, 1985; Niedenthal, 1990). The role of affective schemata in generating specific $S_{ub}L_{it}$ narratives is important on a theoretical level: it seems not only clear that unconscious affective concerns are what generate specific narratives, but more interesting, that somehow affect shapes not only the various cognitive operations (e.g., reversals) and use of specific words and phrases but the specific use of the components of language (i.e., syntax and phonology) as well.

In support of the further study of verbal narrative in cognitive science, Jerome Bruner (1990) has observed: "One of the most ubiquitous and powerful discourse forms in human communication is *narrative*" (p. 77). Cognitive science tends to ignore narrative data, especially the oral aspects, treating these as "written language" (see Haskell, 2002a). Unlike written or more formal language expression, however, oral narrative is an older form of human communication and thus most likely connects to more "primitive," unconscious, and innately evolved neural circuitry. Oral narrative is also more spontaneous, and seems to rely on different cognitive processes than written language (Chernigovskaya, 1994; Luria, 1976; Ong, 1982).[30]

An aspect of logico-mathematic methodology pertains to ecological validity (Neisser, 1976). While establishing validity of the phenomena requires controlled conditions, findings equivalent to those generated under laboratory settings are also observed in everyday narratives.

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[30]An interesting implication of sub-literal material concerns its possible use in therapeutic settings. Independently noticing rudimentary sub-literal-type narratives in therapy sessions, Langs (1978, 1983) claims that what he terms "derivative" meaning in narratives can be used therapeutically. Langsian "derivative" meanings, however, are general and largely methodologically uninformed (see Haskell, 1999a, 1999b; see also Smith, 1991, 1999). Langs has developed a new school of psychotherapy based on them. Unfortunately, he needlessly ties psychoanalytic theory to derivative or unconscious meaning. At this point, however, there is little rigorous research to support his therapeutic claims.

Conclusion

One goal in developing the logico-mathematic methodology was to situate sub-literal findings firmly within cognitive science and psycho-linguistic frameworks. From early in the development of the method it was clear that the novel linguistic findings and cognitive operations were too significant to be relegated to psychoanalytic and literary theory, or to discourse analysis. Toward this end, experimental and logico-mathematic methods need to be examined to further develop the methodology. Though future research may show some operations to be incorrect, others modified, and new ones recognized, the fundamental basis of the method and finding will likely remain intact.

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