

New Physical Foundations for Cognitive Science

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Why should the subject of physics arise in a paper ostensibly concerned with cognitive science and evolutionary biology? If we were advocating a new physics of life and mind simply because we cannot devise an explanation of brain function within the framework of conventional physics, it would appear to reveal a fundamental flaw in the paradigm that we are discussing. If cognition is a biological process, and if biology is ultimately reducible to physics, should not physics be sufficient to entail it? In fact, avoiding such an appearance of being “unscientific” motivates many brain scientists to find a way at all costs to couch their explanations of brain behavior in terms of the traditional concepts of physics. Curiously, they do so while failing to appreciate that the fundamental need for new physics is postulated not by the students of the processes of life and mind, but rather by some of the world’s most renowned physicists. In the present paper, I will use the expression “old physics” to include nineteenth century classical physics, general and special relativity, traditional quantum mechanics and chaotic dynamics. I subsume all of these under the umbrella of old physics because, in spite of their differences, they share a set of metaphysical presuppositions. I will argue that some of these suppositions are deeply flawed and that these flaws render old physics insufficient to portray reality coherently, and that abandoning these flawed concepts may provide new and viable theoretical foundations for both biology and cognitive science.

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Introduction: The Old Physics

Physicalism, Reductionism and Determinism

The old physics is premised on several powerful claims. The first is that if we exhaustively understand the laws of physics we should be able to bend the world

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to our will by judicious application of those laws. This has been a foundation of mainstream science for the past 400 years, and the remarkable scientific achievements of those 400 years seem to have provided a spectacular vindication of it. The power of the ideas that lay the foundations for Newtonian mechanics is genuine and cannot be denied.

A fundamental assumption of the old physics is reductionism. In philosophical circles there are a number of different definitions of reductionism. The distinction that is most relevant for the purposes of the present discussion is that between epistemological and ontological reductionism. Epistemological reductionism, or the reduction of knowledge, is the thesis that all descriptions of natural phenomena can be fully replaced with descriptions of their constitutive elements. Ontological reductionism is the notion that the actual behaviors and properties of items are nothing but the behaviors and properties of their components.

Unfortunately, physicists often ignore the distinction between the ontological and epistemological domains. For clarity of discussion we cannot afford to do so. Thus, we will need to consider that the old physics is predicated on both epistemological and ontological reductionism. It is a deep and fundamental presupposition of physics.

Laplace wrote, in what has become a classical description of determinism, that:

We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at any given moment knew all of the forces that animate nature and the mutual positions of the beings that compose it, if this intellect were vast enough to submit the data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom; for such an intellect nothing could be uncertain and the future just like the past would be present before its eyes. (as cited in Young, 2003, p. 29)

Depending on the commentator, either the entire preceding statement or the “vast enough intellect” to which it refers is called “Laplace’s demon.” It subsumes all of the basic metaphysical presuppositions of classical physics. Explicitly, it is a comment on epistemology. It is a claim, in principle, that we can construct a sufficiently large differential equation, and constrain it with a sufficiently detailed list of initial conditions, and from it the “vast enough intellect” could know the movement of every particle in the universe for all time. Implicitly, the statement is an ontological commitment to the notion that reality is absolutely determined. It is presupposed that “the state of the universe” at an instant in time is the effect of “the state of the universe” at the prior instant, and the cause of “the state of the universe” at the subsequent instant. Furthermore, it is presupposed that this causal entailment structure is unambiguous. A specific instance of “the state of the universe,” allows one and only one subsequent “state.”

Bottom-Up Causation

The notion that events are caused from the bottom up is also implicit in Laplace's statement. The universe is presumed to be fully characterizable by a "state," and the state of the universe is presumed to be *identical* to the accumulation of the states of all of its parts. If any of those parts can be fractioned into subparts, then the state of the part is identical to the accumulation of the states of all of its subparts. Since the state of any part at one instant fully causally entails the state of the same part at the next instant, nothing else is needed to bring about its subsequent state. Furthermore, nothing else is needed to entail the state of any collection of parts since the state of the collection is nothing but the accumulation of states of the constituent parts.

The inevitability of bottom-up causation and the exclusion of top-down causation are conventions so deeply held that physicists seldom see the need to explicitly assert them. Nevertheless, the principle that bottom-up causation is fundamental and that top-down causation is forbidden in traditional physics is explicitly stated in the literature on the philosophy of science, for example in Juarrero (1999, pp. 131, 142–144).

It is easy to see why causation of any kind is seldom explicitly mentioned in physics. If the presupposition of physics that aggregates reduce to their components is valid, then a whole is an epiphenomenon, or a byproduct of causation acting on the parts (Juarrero, 1999, p. 21). Thus, as a direct consequence of Newton's physics, causation is typically seen as nothing more than collisions of particles (Juarrero, 1999, p. 23). If causal talk is reducible to talk of collisions, then why not talk about the collisions themselves, and not clutter the discussion with distracting references to cause? Russell is considered to have administered the *coup de grace* to causation, pronouncing it "a harmful relic of a bygone age" (Russell, 1913/1918).

Despite Russell's dismissal, causation is at the heart of ontology, addressing the issue of why reality does what it does. If cause, the "why" of ontology, can be dismissed as a harmful relic of a bygone age then the "what" of ontology quickly follows. This is one reason that physicists typically make no distinction between ontology and epistemology. Although Russell gave this kind of thinking a philosophical imprimatur less than a century ago, it has been common in physics for several centuries. This is clearly evident in Laplace's quote.

Completeness in Principle

Speaking of that "vast enough intellect," what is the limit on the scope of its epistemology? Laplace's demon is not fundamentally limited from knowing everything. Thus, in principle, if Laplace is right, it is possible to construct a largest model of reality.

If this model is complete, it must be isomorphic to reality. Mathematically, this means that it would be possible to construct a one-to-one onto map, $\vartheta: R \rightarrow M$, where in the most general sense R is the set of all the events in reality and M is the set of all the propositions in the model; ϑ carries events in R to corresponding propositions in M . If the map is one-to-one then every event of reality would map to a different proposition in the model. If the map is onto, then the model would contain no propositions that are not the image of some event in reality.

Furthermore the map would be operation preserving. Suppose event A maps to proposition P , event B maps to proposition Q , and event C maps to proposition S [symbolically, $\vartheta(A) = P$, $\vartheta(B) = Q$, and $\vartheta(C) = S$]. Suppose that event A causes event B causes event C [symbolically, the unary operation \Rightarrow on events signifies "causes," or the transformation of one member of the set of events into another member of the set of events, and it is represented $A \Rightarrow B \Rightarrow C$]. Suppose (as is done in composition of permutations) we compose two unary "causes" (\Rightarrow) operations to obtain a binary causal operation [symbolically, $((A \Rightarrow B) \Rightarrow C) = (A \circ B \Rightarrow C)$]. Suppose proposition P implies proposition Q implies proposition S [symbolically, the unary operation \Rightarrow on propositions signifies "implies," or the transformation of one member of the set of propositions into another member of the set of propositions, and it is represented $P \Rightarrow Q \Rightarrow S$]. Suppose we compose two unary "implies" (\Rightarrow) operations to obtain a binary implication operation [symbolically, $((P \Rightarrow Q) \Rightarrow S) = (P \circ Q \Rightarrow S)$]. It follows that $\vartheta(A \circ B) = \vartheta(C) = S = P \circ Q = \vartheta(A) \circ \vartheta(B)$, or the map preserves the operation.

If a map is one-to-one, onto, and operation preserving, then it is isomorphic. If two processes are isomorphic, one can just as easily refer to one or the other with no loss of understanding. If reality and the largest model of reality are isomorphic, one can just as easily refer to one as the other. The presumed existence of a largest epistemological model of reality lends legitimacy to Russell's advice that we ignore causation.

It is worth mentioning that the largest model that Laplace had in mind would "condense" the entire epistemology to "a single formula." That single formula is a map $\sigma: R^m \rightarrow R^n$, where σ carries members of R^m , the set of m -dimensional vectors of irrational numbers to members of R^n , the set of n -dimensional vectors of irrational numbers. This idea that the whole world is reducible to numbers goes back at least as far as Pythagoras (Guthrie, 1962, p. 230) and remains at the heart of physics. As Lord Kelvin (1883) reminded the Institution of Civil Engineers:

When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the state of science.

It was Lord Kelvin who also articulated the notion of the completeness of the largest model. In a lecture to Royal Institute in 1900 he expressed the commonly held belief among physicists of the time that not only was physics complete-in-principle but was very close to being complete-in-fact. He did note that two small clouds remain over the horizon, the experiments of Michelson and Morley, and blackbody radiation (see Thompson, 1900a).

Recapitulating the Foundational Premises of Old Physics

Old physics is the set of descriptions of the real world based on these axioms:

1. The behavior of the whole is no more than the sum of the behaviors of its parts.
2. The state of the universe at any instant is the effect of its state at the prior instant, and the cause of its state at the subsequent instant.
3. Causation is unambiguous; one and only one state can be caused by a specific immediately prior state.
4. Events in reality are exclusively the effect of bottom-up causation.
5. There is, in principle, a complete or largest model isomorphic to reality.
6. Considering the largest model epistemologically, and implicitly ignoring the causal entailment structure and ontological effects being modeled, results in no loss of understanding.
7. The largest model is a differential equation whose solution is a map, $\sigma: \mathbb{R}^m \rightarrow \mathbb{R}^n$.

Old Physics is Newer than We Think

Why Did the Old Physics Evolve?

Did the “clouds over the horizon” of which Lord Kelvin spoke led to a revolution in physics? In fact, they only slightly changed the premises listed above. At its foundation, modern physics is startlingly similar to classical physics. Although the fundamental premises of physics have been left mostly intact by twentieth century innovations, those innovations seemed radical at the time they occurred. Why do radical innovations arise in a science that had been backed up by centuries of successful prediction? What drove these innovations were the seemingly inescapable paradoxes that arose in physics, and the elegance of the ideas that resolved them.

The Universal Speed Limit

The first major paradox was the discovery of the “universal speed limit.” Although often attributed to Einstein, the phenomenon was actually discovered

decades earlier. Einstein's invaluable contribution was that he resolved the seeming paradox that arose from it.

The phenomenon first arose as a theoretical consequence of Maxwell's equations. Those equations describe the relationship between electric and magnetic fields. The two fundamental equations are known as the "curl" equations:

$$\begin{aligned}\nabla \times \mathbf{E} &= -\partial \mathbf{B} / \partial t \\ \nabla \times \mathbf{B} &= \mu(\mathbf{J} + \partial \mathbf{E} / \partial t)\end{aligned}$$

\mathbf{E} represents the electric field, and \mathbf{B} represents the magnetic field.

It is a straightforward homework problem for third-year engineering students to substitute either curl equation into the other, manipulate a few vector identities, derive a relationship known as the "wave equation," demonstrate that "traveling waves" satisfy the wave equation, and compute the speed of the propagating wavefront. The resulting solution is astoundingly simple (Hecht, 1987, pp. 39–43).

$$c = (\mu_0 \epsilon_0)^{-1/2}$$

c , the speed of an electromagnetic wave (including light) traveling through free space, depends upon μ_0 , the permeability of free space (the ratio of magnetic flux to magnetic field), and ϵ_0 , the permittivity of free space (the ratio of electric flux to electric field), and *absolutely nothing else*.

The solution is not merely astoundingly simple; it is frighteningly simple. The quantities μ_0 and ϵ_0 , are universal constants. This means that the speed of light in free space is a constant. It is independent of location in space, direction of propagation, frame of reference, or anything else. What was most disturbing to nineteenth century physicists was the discovery that c is independent of the motion of any platform from which the propagating light wave might be launched. To appreciate how bizarre this result is, compare it to two motorcycles. Suppose that motorcycle A is on a railroad embankment and moving 60 miles/hour in a straight line parallel to the railroad. Suppose motorcycle B is on a flatcar in a train and moving 60 miles/hour in a straight line along the bed of the car toward the engine. Suppose that the train is moving 80 miles/hour in a straight line along the railroad in the same direction as motorcycle A. If we ask how fast motorcycle B is moving relative to the ground, the answer is 140 miles/hour, and 80 miles/hour faster than motorcycle A, exactly the difference in speed induced by the moving platform. This is the common sense answer, fully in accord with Newton's laws of motion, and if it is tested with real motorcycles and a train, the test would validate the answer.

However, if Maxwell is right, the same experiment will not work if spotlights are substituted for motorcycles. Suppose that spotlight A is affixed to the embankment and the light wave emanating from it is moving 186,000

miles/second in a straight line along the railroad. Suppose that spotlight B is affixed to the flatcar and the light wave emanating from it is moving 186,000 miles/second toward the engine. Suppose that the train is moving 93,000 miles/second (half the speed of light) in a straight line along the railroad in the same direction as the lightwaves emanating from spotlight A. If we ask how fast the light emanating from spotlight B is moving relative to the embankment, Maxwell says that the answer is 186,000 miles/second, exactly the same as the light emanating from spotlight A; the motion of the platform makes no difference.

This answer flies in the face of common sense, defies Newton's well-tested laws of motion, and seems to point to a fundamental flaw in Maxwell's equations. To understand the character of light propagation, Maxwell devised an experiment that many people expected would debunk his bizarre prediction. The Earth moves in its orbit at approximately 1/10,000 the speed of light. If light speeds add directly, then the speed of light propagating in the direction parallel to the movement of the Earth should be about 1.0001 times faster than the speed of light propagating in the direction perpendicular to the movement of the Earth. Michelson and Morley performed the experiment using an apparatus capable of measuring the difference in the speeds of the light beams with a resolution on the order of than 1 in 10,000,000. Measuring the speed of light beams moving parallel and perpendicular to the movement of the Earth, they found no difference. Maxwell's prediction that the speed of light is independent of the motion of the platform from which it is launched was observed in reality (Hecht, 1987, pp. 382–385).

Relativity Is Not New Physics.

Special Relativity: Correcting a Hidden Assumption

What had been dismissed as a small cloud was a genuine paradox. Newton and Maxwell seemed to contradict each other, and their seemingly contradictory predictions were confirmed by well-founded experiments. Einstein's contribution was to identify a false premise and so resolve the paradox. To do so he asked, "Can we conceive of a relation between place and time of the individual events relative to both reference-bodies, such that every ray of light possesses the velocity of transmission c relative to the embankment and relative to the train? This question leads to a quite definite positive answer, and to a perfectly definite transformation law for the space-time magnitudes of an event when changing over from one body of reference to another" (Einstein, 1920, chapter XI, paragraph 2).

Einstein challenges only one premise of the Newtonian tradition. He cannot ignore the fact that both theory and experiment show the speed of light to be independent of the movements of any observer. The speed of light is

fixed, and cannot flex in order to fit into fixed frames of reference that result from assuming that space and time are rigid. However, there is nothing sacred about the assumption that space and time are rigid. If the speed of light is fixed, but propagating light fits into whatever frame of reference it finds itself, might it be the case that the structure of space and time flex in response to disturbances such as propagating light?

This was Einstein's breakthrough. Assuming the answer to be yes, that the structure of space and time flex in response to disturbances, and performing some tedious mathematics to trace out the consequences he arrived his theory of special relativity. Other than reversing the Newtonian hypothesis that velocities are always flexible and the structure of space and time is always fixed with a new hypothesis that some velocities are fixed and the structure of space and time is flexible, Einstein suggested *no other changes* to the Newtonian paradigm. As radical as his idea seemed, it does not merely leave classical physics intact; it strengthens it by correcting a mistake that Newton had no practical way of knowing that he had made.

Relativity Is Not New Physics.

General Relativity: Reinterpreting a Classical Assumption

Einstein's other big idea flowed directly from his first. Special relativity shows that the flexibility of the structure of space and time entails a reality that accommodates the properties of electromagnetism as described by Maxwell's equations, which in turn describe a causal entailment structure that in effect answers the "why" questions about electromagnetic effects. Newton had hypothesized the existence of gravity, and even described it in useful detail, but his description was strictly limited to "what" and not "why" questions. Einstein wondered if the flexibility of the structure of space and time might account for why gravity behaves as it does (Einstein, 1920, chapter XIX, paragraph 1).

It took a further decade to uncover the answer, but he did answer "why" to Newton's "what." The key insight was taken *directly and intact* from classical physics that "the *gravitational* mass of a body is equal to its *inertial* mass" (Einstein, 1920, chapter XIX, paragraph 5). Einstein did not form a new hypothesis; he reinterpreted an old one given the novel insight that the structure of space and time is flexible. "It is true that this important law had hitherto been recorded in mechanics, but it had not been *interpreted*. A satisfactory interpretation can be obtained only if we recognize the following fact: *The same* quality of a body manifests itself according to circumstances as 'inertia' or as 'weight'" (Einstein, 1920, chapter XIX, paragraph 6). The presence of a mass warps the fabric of the surrounding space, and the acceleration induced by the warp is experienced as gravity (Einstein, 1920, chapter XXIX).

No Foundational Presuppositions Overturned

Special and general relativity have led to amazing predictions of bizarre behaviors that have been observed in reality against intuitive expectation, but they do not constitute a new physics. Einstein changed none of the seven fundamental premises of physics listed above. In special relativity Einstein corrected a logical inconsistency in classical physics by introducing the notion that the structure of space and time is flexible. General relativity amounts to *the reinterpretation of the pre-existing classical insight* of the equivalence of inertial and gravitational mass. Everything else follows inevitably from the mathematical analysis of these concepts.

The theories of relativity do not constitute a claim that “everything is relative,” as they are often incorrectly characterized. Einstein required several indispensable absolutes, first, that the speed of light in free space is unconditionally constant, and second, that there is only one kind of acceleration, irrespective of its cause. He then described a coherent reality that could rest upon those absolutes. It in no way diminishes the magnitude or the grandeur of the feat that Einstein accomplished to observe that instead of devising a new physics, he showed what classical physics could do if a single incoherent premise were corrected.

The Ultraviolet Catastrophe

The other “cloud over the horizon” was blackbody radiation. If solid objects are heated, they emit visible light. As the temperature changes the perceived color changes from red hot, through orange hot to white hot. The spectrum of the light emitted by a hot solid object depends on the temperature. The relationship between the temperature and the emitted spectrum is similar for all solid bodies. The idealized relationship between the temperature and the emitted spectrum of a hot solid body is called blackbody radiation. It is not particularly difficult to construct an object that closely approximates the behavior of a blackbody (Sproull, 1966, pp. 108–112).

Ironically, the difficulty arises from the fact that the properties of blackbody radiation are easy to observe experimentally. The observation raises an unmistakable red flag: only in the deep infrared does the spectrum even remotely resemble what is predicted by classical physics. As the wavelength moves toward the ultraviolet, classical physics predicts that the blackbody radiation should become more and more intense, seeming to approach infinity. As the wavelength moves toward the ultraviolet, the experiment reveals that the blackbody radiation actually becomes less and less intense, seeming to approach zero. Decades of effort have been spent trying to find an explanation of this behavior within the bounds of classical physics. All have failed. This

failure to explain blackbody radiation at ultraviolet wavelengths suggests a fundamental flaw in classical physics, and is called the “ultraviolet catastrophe.”

The Solution Leads to Bigger Problems

Planck discovered a new theory that describes blackbody radiation. The theory from classical physics presumes that the light is generated by an assembly of oscillators within the blackbody, with some oscillators at every wavelength throughout the spectrum. Planck retained the presumption that the light is generated by an assembly of oscillators, but hypothesized that the oscillators operate at a discrete sequence of wavelengths throughout the spectrum with oscillations at any other wavelengths being disallowed. That presumption of quantization of wavelengths led to an equation that agreed with the observed data astoundingly well, and thus began the radical new field of quantum mechanics.

However, there is no physical reason for the hypothesis of quantization of wavelengths besides the fact that the resulting curve that fits the empirical data. Planck very inventively answered the “what” question, but totally ignored the “why” question. This was to become the paradigm for quantum mechanics. Starting from Planck’s concept of quantization, Schrödinger subsumed the entire theory of quantum mechanics in a single equation describing the location of a particle in an energy field. In normalized units it has the following form:

$$\nabla^2\Psi - P\Psi = \partial\Psi/\partial t$$

In Schrödinger’s equation, t represents time, P represents energy as a function of location, but what is Ψ ? $|\Psi(x, y, z)|^2$ is the probability of finding the particle at the coordinates (x, y, z) [Sproull, 1966, pp. 140–141]. Does Ψ itself have no physical meaning? It is a “wave function” with the mysterious property of “collapsing” if we measure the coordinates, (x, y, z) . Does the act of measuring (x, y, z) cause some physical aspect of the particle to collapse? Perhaps it is only a bit of our ignorance that collapses when we know the position of the particle.

Quantum Mechanics Is Not New Physics

Quantum mechanics is not classical physics, and there is no a priori physical justification for the hypothesis of quantization that leads to Planck’s or Schrödinger’s equations. However, this is a difference in detail, not in fundamental presuppositions. The presuppositions of quantum mechanics remain strikingly similar to classical physics. This is most apparent in the seventh presupposition. The entire model of quantum mechanics is subsumed in Schrödinger’s equation, a differential equation whose solution is a map, $\Psi: R^m \rightarrow R^n$.

The sixth presupposition of classical physics considers its model epistemologically, and *implicitly* ignores the causal entailment structure and ontological effects being modeled. The corresponding presupposition of quantum mechanics is subtly different. It also considers its model epistemologically, but *explicitly* ignores the causal entailment structure and ontological effects being modeled. This is obvious from the meaning of $\Psi(x)$; it is the complex square root of the probability of finding a particle between the locations x and $x+\delta x$. In classical physics, Maxwell's equations are fundamentally ontological, making a comment on the electromagnetic field itself. In quantum mechanics, Schrödinger's equation is fundamentally epistemological, making a comment on our *knowledge* of the location of a particle.

The Uncertainty Principle: Strictly Epistemological

The fifth presupposition of classical physics asserts that there is, in principle, a complete or largest model isomorphic to reality. Clearly, the uncertainty principle implies that a map between events in reality and propositions in the model cannot be assured of being 1 to 1 or onto. The isomorphism breaks down.

However, this turns out *not* to be a difference between quantum mechanics and the physics of the world of macroscopic sizes. It is a little-appreciated fact that both theories share the uncertainty principle. It is well known that Heisenberg formulated the principle that $\Delta x \Delta v = \text{constant}$, meaning that we can *know* either the position or momentum of a particle with as much precision as we like, at the expense of our *knowledge* of the other (Sproull, 1966, pp. 122–129). What is less well known is that various researchers have discovered that the same principle applies at the macro scale; Gabor's development of a macro-level uncertainty relationship has had a significant impact on recent strategies for signal processing (Gabor, 1946).

In either case, the uncertainty principle is a comment on epistemology and *not* ontology. It states a limit on how much we can learn about a particle or a signal by taking a measurement. It is *not* a description of a constraint on the actual dynamics of the particle or signal. Since the uncertainty principle is a comment on knowledge, and not on the physical process that is the subject of that knowledge, it is not a denial of the causality of the process.

Heisenberg's comment on the uncertainty principle was a declaration that causation is irrelevant rather than non-existent.

In view of the intimate connection between the statistical character of the quantum theory and the imprecision of all perception, it may be suggested that behind the statistical universe of perception there lies hidden a "real" world ruled by causality. Such speculation seems to us — and this we stress with emphasis — useless and meaningless. For physics has to confine itself to the formal description of the relations among perceptions. (Heisenberg, 1927, p. 197)

Notice how he frames the argument: “physics has to confine itself to the formal description.” In other words, Heisenberg asserts that physics is confined to epistemology. If an ontological argument can show that causation is necessary for the relations among those imperfectly calibrated perceptions to make sense, then no matter how compelling the argument might be, it is nevertheless to be dismissed as a useless and meaningless speculation, for the overarching reason that it is not within purview of physics as Heisenberg defined that purview.

Ignoring Rather than Denying Causation

The fourth, third and second presuppositions of classical physics are all concerned with causation. From Heisenberg’s comment, quantum mechanics ignores causation, and suggests that such self-imposed ignorance has the same force as denial. However, none of these ontological presuppositions is overturned by quantum mechanics; instead the propositions are explicitly ignored.

The first presupposition of classical physics is reductionism: whole is the sum of the parts and nothing more. The difference between classical and quantum physics on reductionism is trifling. Classical physics is concerned with epistemological reductionism, and *implicitly* ignores ontological reductionism, supposing that the two forms of reductionism are isomorphic. Quantum mechanics is concerned with epistemological reductionism, and *explicitly* ignores ontological reductionism, supposing that ontology is outside the scope of physics.

Like relativity, quantum mechanics has led to amazing predictions of bizarre behaviors that have been observed experimentally against intuitive expectation. However, at the fundamental level, like relativity, it does not constitute a new physics. The only one of the presuppositions of nineteenth century physics actually invalidated by a twentieth century insight is the notion that there exists a model isomorphic to a physical process. This is not an instance of quantum mechanics striking at the heart of classical physics. Gabor derived the uncertainty principle on the macro scale from classical principles, thereby showing that the possibility of a complete model is not one of those principles. In quantum mechanics, the other six presuppositions of classical physics are either simply ignored or are recast in epistemological terms.

Chaos Theory

One other idea that has been trendy in recent years is chaos theory. It is important to appreciate what chaos theory actually involves. Both classical and quantum physics share the explicit claim that all the description worth mentioning can be captured in a differential equation. Until about twenty years ago, both classical and quantum physics implicitly shared the even nar-

rower claim that *everything that matters* can be captured in a *linear* differential equation.

The distinguishing feature of a linear system is the principle of superposition. If O is a linear operator, a and b are constants, and f and g are functions, then $O(af + bg) = aO(f) + bO(g)$. If O is not linear, then this does not hold. For example, $O(af + bg) = h$ where h can have all sorts of strange dependencies on a , b , f , and g . An example is $h = a^2 + abf^3\sin(abfg)$. Linear systems are popular among engineers and scientists because the differential equations that characterize them are convertible into easier-to-solve algebraic equations.

Within a particular range of properties of a non-linear system, given a single periodic input, the output will have a broadband Fourier spectrum; this is deterministic chaos (Thompson and Stewart, 1986, p. 25). There is nothing particularly remarkable about a non-linear system producing chaotic response to a periodic input. It is as fully determined as the periodic response of a linear system to a periodic input. Thompson and Stewart show that the constraint entailed by non-linear determinism is seen in the fact that "For Hamiltonian, energy-conserving systems, the Liouville Theorem states that the volume occupied by any ensemble of states (points) in phase space remains constant as the ensemble evolves in time" (1986, p. 221).

A chaotic response exhibits extreme sensitivity to initial conditions. This has a specific mathematical definition: for a given system and two slightly different initial conditions the average separation between the responses increases by a fixed multiple for any given interval of elapsed time (Thompson and Stewart, 1986, p. 4). In other words, there is exponential divergence between the responses. Nevertheless, as long as the initial conditions are the same for every trial, exactly the same chaotic output is obtained every time.

Extreme sensitivity to initial conditions does not mean that chaos is equated with blithering confusion. The differing responses to different initial conditions occupy a bounded region of phase space (Thompson and Stewart, 1986, p. 94). This means that we *do not* require infinitely precise knowledge of initial conditions to make a reliable prediction about the state of the chaotic system. For an arbitrary time in the future, to limit the prediction error within a specific bound, there is another specific bound on the range of initial conditions that we can specify.

This is a severe restriction, and it is practical to meet only under certain conditions. If we linearly decrease the bound for the allowable error of the prediction, then we must improve the precision of our estimate of the initial conditions exponentially. Nevertheless, the only circumstance under which we would require infinitely precise knowledge of initial conditions would be if we were trying to estimate the state of the system into a limitless future.

Predicting Chaos Is Difficult but Not Impossible

It is widely supposed that extreme sensitivity to initial conditions overturns Laplace's demon (Gleick, 1987, p. 14). It does not. All it means is that the demon cannot look infinitely far into the future. Since only a finite amount of time remains until the "big crunch," the "vast enough intellect" may take a finite look into that future as far as it likes. Since it must know the initial conditions with a precision that increases exponentially with the distance it looks into the future, the intellect may need to be vaster than Laplace envisioned. Nevertheless, as long as the look is to a time finitely far into the future, if the intellect is vast enough it can get the job done.

An intellect might be vast enough in principle, but not in practice. Human-made processes must be safe and reliably controllable without the necessity for an impractically vast intellect. To this end, much of the work of engineering is concerned with the design and fabrication of components with unnaturally precise dimensions, extreme material purities and narrow operating ranges, all with the specific intention that they should closely track the description afforded by linear differential equations, and thus be easy to predict and control.

In contrast, processes that can be adequately characterized by linear differential equations seldom occur in nature (Gleick, 1987, pp. 67–69). Chaos theory is characterized by non-linear differential equations. A key consequence is that effects previously dismissed as noise or experimental error are now understood to be inherent to the process. The fact that these predicted effects are real and inherent properties of natural systems was only grudgingly admitted after it became clear that they were too important to be ignored.

Chaos Is Not New Physics

Chaos is not a new physics: it is the old physics done with the decision not to ignore those equations that were supposed to be hard to solve. This is most apparent in the seventh presupposition. The entire model of a chaotic system is a differential equation whose solution is a map, $\phi: \mathbb{R}^m \rightarrow \mathbb{R}^n$. Chaos theory simply imposes the perfectly reasonable requirement that the non-linearities in the equation not be ignored.

The sixth presupposition of classical physics considers its model epistemologically, and *implicitly* ignores the causal entailment structure and ontological effects being modeled. The corresponding presupposition of chaos theory is identical. This is seen in the widespread lamentations over the fact that there is a limit to what we can know about a process. However, as Gabor shows in the uncertainty principle for macroscopic processes, there is a limit to what we can know even about linear systems.

The fifth presupposition of classical physics, unchallenged by chaos theory, asserts that there is, in principle, a complete or largest model isomorphic to reality. The non-linear differential equation is claimed to be the complete description of the causal entailment structure of the chaotic system. The requirement for an exponential precision of initial conditions, if we wish to use that differential equation to make a reliable prediction about the chaotic system, is more restrictive than the precision required for linear systems, but is no different in principle.

The fourth presupposition of classical physics is that events in reality are exclusively the effect of bottom-up causation. This is unchanged by chaos theory. The non-linear differential equation is still a description of the effect of the parts on the whole. As a typical differential equation in classical physics it ignores the effect of the whole on the parts.

The third presupposition of classical physics is that causation is unambiguous; one and only one state can be caused by a specific immediately prior state. The fact that a chaotic attractor does not intersect itself in phase space is simply a restatement of this very principle.

The second presupposition of classical physics is that the state of the universe at a given instant in time is the effect of the state of the universe at the immediately prior instant in time, and is the cause of the state of the universe at the immediately subsequent instant in time. As a comment on the nature of reality, this is the same for both linear and non-linear processes. Extreme sensitivity to initial conditions does not change that principle. The present state is the exact initial condition for the one and only one possible next state. The fact that there are limits on the precision of our knowledge of both states does not change the underlying reality.

The first presupposition of classical physics is reductionism. Contrary to some comments in the literature, chaos theory overturns neither epistemological nor ontological reductionism. The non-linear differential equation of chaos is derived from considering the parts and not the whole; it is a more detailed description of the interaction of the parts than one obtains by approximating the non-linear process by a linear equation. The "whole" is nothing but the aggregation of the interactions of the parts; this is the reductionist paradigm of classical physics. Chaos simply considers more of the interactions between the parts than linear systems theory does.

Like relativity and quantum mechanics, chaos theory has led to amazing predictions of bizarre behaviors that have been observed experimentally against intuitive expectation. Nevertheless, at the fundamental level, like relativity and quantum mechanics, it does not constitute a new physics. None of the presuppositions of classical physics is overturned by chaos theory. In fact, it is nothing but classical physics with the non-linearities taken into account.

What's Wrong with This Picture?

Paradoxes from the Ontology/Epistemology Confusion

The foundational concepts that Newton (1687/1999) proposed had undergone two centuries of rigorous philosophical analysis and experimental validation when Kant pronounced them to be “true and irrefutable,” a view shared by Poincaré, one of the most respected philosophers living at the turn of the twentieth century (Magee, 1997, p. 187). Although Lord Kelvin noted “clouds over the horizon,” they clearly impressed him as insignificant because he also observed, “There is nothing new to be discovered in physics now. All that remains is more and more precise measurement” (Thompson, 1900b).

Despite the fact that almost all of Newton's presuppositions continue to serve as the foundation of physics even to the present day, at the turn of the twentieth century two of Newton's time-honored assumptions were being undermined. In 1901 Max Planck found it necessary to challenge the notion of continuity of states: “Moreover, it is necessary to interpret U_N not as a continuous, infinitely divisible quantity, but as a discrete quantity composed of an integral number of finite equal parts” (Planck, 1901, p. 553). Likewise, in 1905 Einstein replaced a classical absolute, “. . . the view here to be developed will not require an ‘absolutely stationary space’ provided with special properties . . .” with an alternative absolute. “Light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body” (Einstein, 1905, p. 891). Within about two decades quantum mechanics was derived from the first insight, and relativity from the other, and both became widely accepted as part of mainstream physics.

Why was there such an abrupt turnabout? Why had concepts that had withstood centuries of scrutiny lost their power seemingly overnight? Clearly, concepts that had stood the test of time could only be swept away by the severest of challenges. Quantization was the answer to the “ultraviolet catastrophe,” and the flexure of space–time was the answer to the “universal speed limit.” In other words, the paradigm shifts arose because Planck and Einstein discovered compelling concepts that resolved seemingly overwhelming paradoxes.

Paradoxes continue to arise in physics (Jaynes, 1989). Most arise from the failure to distinguish the ontological from the epistemological. As already noted, this failure to distinguish the model from the process being modeled has its roots in the Newtonian/Laplacian notion that a complete description of reality is possible in principle, and if the description is isomorphic to reality we can save a great deal of difficulty by focusing on the model and ignoring the reality.

It is a twentieth century discovery that the isomorphism breaks down. The uncertainty principle, as recounted in quantum mechanics by Heisenberg and

in macroscopic processes by Gabor, precludes the possibility of a one-to-one correspondence, and consequently isomorphism, between events in reality and propositions about them. There are two ways to respond to this limitation. One is to realize that the inferential entailments of any given model constitute an informed guess about causal entailments in the underlying reality and that if a model fails to explain an observed event in reality, that failure should cause us to seek more information in order to make a more informed guess. The other response is to ignore the breakdown in the isomorphism, and continue to *assume* that the properties of our models are properties of reality.

Consider the common interpretation of the uncertainty principle. The indeterminism of the quantum mechanical model implies that our *knowledge* of reality is unentailed by events in reality. Heisenberg asserts that "physics has to confine itself to the formal description," i.e., is strictly epistemological. Thus, he dismisses questions about causal entailment as "useless and meaningless." From there, it is but a small step to suppose that if causal entailments in reality are meaningless, then they do not exist. This seems to put the imprimatur of physics on Russell's philosophical claim that the notion of causation is "a harmful relic of a bygone age."

The problem is that the interpretation does not stop with Heisenberg's admonition to refrain from asking questions about causal entailment because they are beyond the scope of quantum mechanics. All that the uncertainty principle tells us is that *our knowledge* of the position, x , and momentum, p , of a particle dances about willy-nilly inside a window whose size is $\Delta x \Delta p$. The common interpretation states that our uncertain knowledge somehow drives a particle in reality to behave that way. For example, Greene claims: "Heisenberg's Uncertainty Principle asserts that a similar frantic shifting back and forth of energy and momentum is occurring perpetually in the universe on microscopic distance and time intervals" (1999, p. 119). Heisenberg asserted no such thing; if such a "frantic shifting back and forth" were occurring inside a window of uncertainty, Heisenberg said that physics would be powerless to identify it.

The error in thinking reflected in the quote from Greene is typical of a class of errors that Jaynes calls the "mind projection fallacy." The fallacy is the supposition "that one's own ignorance signifies some kind of indecision on the part of nature" (Jaynes, 1989, p. 7). Greene's quote illustrates Jaynes's observation that "the current literature of quantum theory is saturated with the mind projection fallacy."

It is tempting to ask if this is really a problem. If as Haldane (1928, p. 286) suggests, "the universe is not only queerer than we suppose, but queerer than we can suppose," is it unreasonable to suppose that the indeterminism of our knowledge projects disentanglement onto the structure of reality, "as if one were to

control Nature by psychokinesis?" (Jaynes, 1989, p. 7). It is unreasonable. Jaynes shows that, even for simple physics problems, it leads directly to paradox.

Jaynes's example of diffusion is instructive. Suppose one pours a spoonful of sugar into a big pot of water. The sugar falls into the pot in a concentrated cluster of crystals and dissolves, quickly resulting in a high concentration of sugar molecules near the location of the pour, initially with no sugar molecules located near the edges of the pot. If the pot is left to sit for long enough, the sugar diffuses, resulting in a near-uniform concentration of sugar molecules throughout the water in the pot.

Observing the process raises the question of how quickly the diffusion occurs. A common strategy to find an answer is based on the observation that from early-on in the diffusion process, each sugar molecule is completely isolated from all the other sugar molecules and that its movements are all the result of its collisions with neighboring water molecules. Jaynes demonstrates that this strategy leads to a prediction that on the average the sugar particles do not move and that diffusion never occurs. In reality, diffusion is seen to occur. The problem arises from the hidden assumption that the probability of future movement is an ontological property of the molecule. This is an instance of the Mind Projection Fallacy and it gives rise to the paradox.

The paradox is resolved by recognizing that the probability of future movement is not an ontological property of the molecule, but rather is an epistemological comment on what we can expect from the information that we decide to take into account. The movement of the sugar molecule is *not* unaffected by its prior location and density simply because we choose to ignore them. If we consider the probability of movement to be our best guess based on what we know, we can add more information and improve the guess. In the case of diffusion, we have the prior information that the concentration was $n(z)$ when the molecule was located at z . The drift velocity is computed by assuming that the particle is located at position x , and asking what is the probability that it was at position z at a given small interval of time, τ , in the past, and using the fact that the concentration was $n(z)$ at that moment in the past. By a straightforward application of Bayes's theorem, the drift velocity is found to be $v = -(\delta x)^2/2\tau V(\log(n))$, exactly as observed experimentally. Eliminate the mind projection fallacy, and the paradox vanishes.

How are we to keep the mind projection fallacy from confusing our thinking? This is difficult since it often intrudes upon the reasoning process via hidden assumptions, premises tacitly hypothesized but not acknowledged (or even suspected). Clarity of thought requires that we keep in perspective both our need and our capacity to make decisions on the basis of incomplete information. "We are hardly able to get through one waking hour without facing some situation (e.g., will it rain or won't it?) where we do not have enough information to permit deductive reasoning; but still we must decide immedi-

ately what to do" (Jaynes, 2003, p. 1). Jaynes says that we reach such necessary but partially-informed decisions by *plausible reasoning*.

Most crucially, Jaynes sees the proper role of probability as a tool for facilitating the process. He starts with three desiderata of plausible reasoning: (1) degree of plausibility is represented by an irrational number; (2) plausible reasoning should correspond qualitatively to common sense; (3) it should be consistent. From these desiderata he derives the principles of probability theory. Speaking of the quantities p , in his calculations, Jaynes says, "They define a particular scale on which degrees of plausibility can be measured." In other words, a probability is a measure of what we know (2003, p. 37).

Plausible reasoning is a generalization of logic. Jaynes notes that "Aristotelian deductive logic is the limiting form of our rules for plausible reasoning" (Jaynes, 2003, p. 31). That limit is complete information. In the absence of complete information, plausible reasoning serves as an algorithm for making the best possible guess with whatever information is available.

For simple problems he gets the same formulas as Bernoulli and Laplace got by the *ontological* strategy of counting balls in urns. However, Jaynes arrived at those formulas by a significantly different strategy. Noting that probability is an *epistemological* comment, he says, "The important new feature was that these rules were now seen as uniquely valid principles of logic in general, making no reference to 'chance' or 'random variables'" (Jaynes, 2003, p. x).

This has significant consequences for reasoning about physical processes. There is no requirement that probability be correlated with random outcomes in ontological reality. Probability is a measure of the plausibility of our description of a situation. We are not justified in assuming that our ignorance of a process implies random behavior in the process. When we make that assumption, it frequently leads to paradoxes.

This has a profound implication for new physics. Jaynes's discoveries about the role of probability invalidate the sixth premise of traditional physics. It is not the case that considering a model epistemologically, and implicitly ignoring the causal entailment structure and ontological effects being modeled, results in no loss of understanding.

If there is a need for new physics, such a physics must do away with the fifth and sixth premises. Instead it must be based on the notion that there is no largest or complete model of a process in reality, and not even Laplace's "vast enough intellect" will find one. More crucially, our knowledge, or ignorance, does not drive ontological randomness. Any genuine understanding of reality must ask questions about the causal entailments of reality as well as questions about the inferential entailments of our models of reality. What Jaynes's resolution of the paradoxes that flow from the mind projection fallacy shows is that it is the *dismissal* of questions about causation that has now become the "harmful relic of a bygone age."

The Need for Superphysics

The discoveries of Heisenberg, Gabor, and Jaynes lead to only a slight revision in the original presuppositions of physics. Taking into account these revisions, we can still say that the strategies of classical physics, special relativity, general relativity, quantum mechanics and chaos theory either specifically presuppose or ignore (but do not prohibit) the following presuppositions:

1. The behavior of the whole is the sum of the behaviors of the whole's parts and nothing more.
2. The state of the universe at a given instant in time is the effect of the state of the universe at the immediately prior instant in time, and the cause of the state of the universe at the immediately subsequent instant in time.
3. Causation is unambiguous; one and only one state can be caused by a specific immediately prior state.
4. Events in reality are exclusively the effect of bottom-up causation.
5. All the available information about a physical process can be described by a differential equation whose solution is a map, $\sigma: \mathbb{R}^m \rightarrow \mathbb{R}^n$.

Any revision to physics that leaves these presuppositions intact is simply an update of the old physics. Any body of thought sufficiently radical that we might seriously consider it to be new physics must overturn one or more of these presuppositions. Are we particularly keen to do so? These presuppositions have served us amazingly well for many centuries. Any reason for considering the need for a new physics must be as profound as those that motivated Planck, Einstein, and Jaynes to question some of the old presuppositions. We need new physics if and only if the old physics leads to paradox.

The most publicized paradox is that quantum mechanics and relativity contradict each other. As Greene (1999, p. 129) says, "The notion of a smooth spatial geometry, the central principle of general relativity, is destroyed by the violent fluctuations of the quantum world on short distance scales." At first glance, this seems a classic illustration of Jaynes's idea of paradoxes arising from the mind projection fallacy. In predicting the existence of those "violent fluctuations," Wheeler (1955) says, "Because it is the essence of quantum mechanics that *all* field histories contribute to the probability amplitude, the sum not only may, contain doubly and multiply connected metrics; it must do so" (p. 535). Clearly, Wheeler is asserting that probability is a physical property of fields rather than a comment on what we know about the fields.

If this were all there were to it, we could dismiss "quantum foam," whose existence Wheeler presumed to follow from uncertainty, as a phenomenon *neither required nor forbidden* by the uncertainty principle. Indeed, by Heisenberg's strictly epistemological interpretation of the uncertainty principle it is impossi-

ble to determine whether or not the quantum foam exists and speculation about it is “useless and meaningless.” However, even if we follow Heisenberg’s advice and confine ourselves to epistemology, there remains a paradox. If we combine the equations of quantum mechanics and general relativity, infinities arise, and they are stronger infinities than those that are renormalized away in traditional quantum mechanical computations. The infinities indicate that an epistemology that contains the equations of both quantum mechanics and general relativity is logically incoherent.

The paradox is evident both ontologically and epistemologically. Quantum mechanics and general relativity contradict each other. There is a genuine need, identified by physicists themselves, for a new physics that coherently accounts for both the microscopic effects described by Schrödinger’s equations and the astronomically large effects predicted by general relativity. It is reasonable to expect that such a new physics would explicitly overturn at least one of the five presuppositions listed at the beginning of this section.

Superstrings

A currently popular strategy for resolving the paradox is superstring theory. It started with a successful attempt to explain why the observed properties of the strong nuclear force happen to fit the Euler–beta function (Greene, 1999, p. 137). If one hypothesizes that the elementary particles are vibrating one-dimensional strings, the equations that describe their behavior have solutions in terms of Euler–beta functions. Subsequent work led to the prediction of gravitons, a hint that the theory might reconcile quantum mechanics and general relativity.

Since then, superstring theory has suffered from four difficulties. First, it has progressed by fits and starts; a superstring theory hypothesis solving a specific problem rapidly leads to contradictions when applied to other problems. Although this has progressively led to better hypotheses, each resolving all the previous contradictions, there always seems to be a new contradiction just ahead. Second, superstring theories inevitably require high-dimensional spaces in order to provide coherent solutions; this concern seems more aesthetic than scientific, but it has proven to be a hindrance to the acceptance of the theory. Third, the theory is based on approximate solutions to approximate equations, and many of its predictions are decades away (if ever) from experimental testing. This seems a shaky foundation for what some expect to become a “theory of everything.” Fourth, the superstring theorists exhibit the same “there is nothing new to be discovered in physics” attitude as Lord Kelvin. Superstring literature abounds with comments like “It may well be that there aren’t other surprises” (Greene, 1999, p. 318). The obvious error of this perspective leads one to wonder what other mistakes they might be making.

Although the idea of fundamental particles with finite dimensionality and

the necessity for eleven such dimensions seems radical to mainstream physicists, there is a much more relevant question. Is superstring theory radical enough? Its chief theoretical strategy is the perturbation method, a technique that has proven fruitful over the years in quantum mechanics. In its present form, superstring theory does not appear to challenge any of the five presuppositions listed above. It is not a foregone conclusion that it will fail to realize the expectations of its champions. However, it is so similar to traditional physics in both its presuppositions and its theoretical strategies that it hardly seems to have the makings of a radical paradigm shift.

Physicists on a New Paradigm

Bohm: An Ontological Interpretation of Ψ

Another contradiction between quantum mechanics and relativity arises in quite a different way. Despite Heisenberg's admonition that it should be good enough, it is deeply unsatisfying to settle for the notion that $\Psi(x, y, z)$ signifies nothing more than a complex square root of the probability that we will find the particle at the coordinates (x, y, z) . Bohm departs from this strictly epistemological convention, offering an ontological interpretation of quantum mechanics.

To interpret Ψ ontologically, first consider that it has a magnitude, R . Q is a quantum potential that depends on $\nabla^2 R / R$. Particle motion is described by an updated version of Newton's second law, $m \, dv/dt = -\nabla V - \nabla Q$. "This means that the forces acting on it (the particle) are not only the classical force, $-\nabla V$, but also the quantum force $-\nabla Q$ " (Bohm and Hiley, 1993, p. 30).

Q represents a "quantum field" with very peculiar properties. The amplitude of R appears in both the numerator and denominator of the definition of Q , canceling strength out. The effect of Q depends on the form of Ψ , independent of its strength. It has the power to reorganize energy non-locally. Bohm and Hiley (1993, p. 35) observe that "Such behavior would seem strange from the point of view of classical physics. Yet it is fairly common at the level of ordinary experience. For example we may consider a ship on automatic pilot being guided by radio waves. Here, too, the effect of the radio waves is independent of their intensity and depends only on their form. The essential point is that the ship is moving with its own energy and that the *form* of the radio waves is taken up to direct the much greater energy of the ship."

Ψ is a manifestation of a quantum field that conveys *active information*. As Bohm and Hiley (1993, p. 35) explain, "The basic idea of active information is that a form having very little energy enters into and directs a much greater energy." This fundamentally differs from the passive information in Shannon's information theory. The receipt of passive information merely removes a bit of ignorance from the recipient. Active information entails organized action.

More importantly, that entailment operates non-locally. Bohm and Hiley (1993, p. 38) recognize something significant: "A very important further implication of the notion of active information is that in a certain sense an entire experiment has to be regarded as a single undivided whole."

Non-Locality: An Experiment with Magic Coins

To appreciate how this leads to a paradox in the old physics, Dress (2000) hypothesizes a pair of magic coins. Individually, each coin is fair. Over many individual coin tosses, each coin comes up heads just as often as tails. The "magic" is that if both are tossed at the same time, both coins always land with the same face up. This is a precise analogy to what is observed in quantum entanglement.

Nobody has figured out how to do the trick with macro-sized coins, but it is observed in polarized photons. There is nothing in classical physics that shows anything like this kind of behavior. If an atom is excited in a process known as an "SPS cascade" it will emit two photons moving in opposite directions with exactly the same polarity (Peres, 1995, p. 155). That polarization is unpredictable, just like the coin toss. Suppose a polarization detector is placed in the path of one of the photons a fixed distance from the SPS source. It will register the polarization of the incident photon. Suppose another polarization detector is placed in the path of the other photon a longer distance from the SPS source than the first detector. The second detector will register the polarization of the incident photon a short time after the corresponding reading is registered by the first detector.

The reading by the second detector is always the same as the reading by the first. Despite the fact that the sequence of readings from either detector looks random, the two sets of readings are perfectly correlated. It is as if the first photon somehow reaches out and forces the second to take its polarization.

Why is this reaching out necessary? In quantum mechanics it is presumed that the polarization of a photon is not fixed (and varies randomly) until the act of measurement occurs. The act of measuring the polarization is presumed to fix its value. Suppose the first photon is observed, and thereby acquires the polarization reported by the measurement. The two photons are in a "pure state"; each has the same polarization as the other, whatever that might be (Jaynes, 1989). Consequently, the second photon must also have its polarization fixed by the measurement made on the first, although no measurement is made on the second photon until later, if ever. The second measurement always reports the same polarization as the first.

What Einstein Got Right: Existence of Hidden Variables Revealed

How is the influence of the first measurement transmitted to the second photon? Recall that the second photon is receding from the first at exactly the speed of light, and the two are already a long distance apart by the time the first observation occurs. Suppose, at the instant of the first measurement, the first photon initiates a message telling the second photon what polarization it must take. If the message moves no faster than the speed of light, then it never overtakes the second photon. If the first photon did influence the second, the influence would need to travel faster than light, in violation of the universal speed limit. This is the paradox that arises from entanglement, and it is of the kind suggested by Einstein, Podolsky, and Rosen (1935).

Einstein characterized the influence as the result of the action of "hidden variables." These describe causal entailments not visible to the epistemology of quantum mechanics. (Bohm and Hiley, 1993, p. 19) The wave function describing photon entanglement is exclusively a comment on effect. It says nothing about what produced the effect, neither admitting nor denying the possibility of hidden variables. In fact, the failure of quantum mechanics to take notice of the hidden variables is what caused Einstein to pronounce quantum mechanics "incomplete."

Einstein put one more condition on these hidden variables. He insisted that they should act locally. Non-locality is the notion that two particles can be "strongly coupled over long distances" (Bohm and Hiley, 1993, p. 57). Bohm shows that non-locality violates a key presupposition of the old physics: the behavior of the whole is the sum of the behaviors of its parts and nothing more. In what Bohm calls a "radical departure," he says "the quantum potential, Q , depends on the 'quantum state' of the whole system in a way that cannot be defined as a pre-assigned interaction between all the particles" (Bohm and Hiley, 1993, p. 58).¹

Bohm does not merely suggest overturning the notion of ontological reductionism. He says that something more is involved: "The interaction of the parts is determined by something that cannot be described solely in terms of these parts and their preassigned interrelationships" (Bohm and Hiley, 1993, p. 58). Bohm recognizes an influence of the whole upon the parts: "Something with this kind of dynamical significance that refers directly to the whole system is thus playing a key role in the theory. We emphasize that *this is the most fundamentally new aspect of the quantum theory*" (Bohm and Hiley, 1993, pp. 58–59).

Bohm is speaking of his ontological interpretation of quantum mechanics. The implication of wholeness is fundamentally new. It overturns another pre-

¹In the context of this comment it must be recalled that Bohm is making another radical departure from traditional quantum mechanics: he presupposes that the wave function *has* ontological meaning.

supposition of old physics that says that events in reality are exclusively the effect of bottom-up causation. It is little wonder that Einstein, a believer in the presupposition of exclusive bottom-up causation, dismissed top-down influence as “spooky action at a distance.”

Irrespective of the local versus non-local character of hidden variables, there appears to be a causal entailment structure that is physically real but hidden from the epistemology of quantum mechanics. Peres (1995, p. 158) says that experiments with entangled photons indicate its influence: “The perfect correlation of distant and seemingly random events . . . suggests that the fundamental laws of physics are deterministic, and that the apparent stochasticity of quantum phenomena is merely due to our imperfect methods of preparing physical systems.” Quantum *causation* has been observed.

What Einstein Got Wrong: Non-locality of Hidden Variables

Although we do not know the character of these hidden variables, it is possible to construct a model of them and identify some limitations on what properties might be required of or forbidden to them. Bell presupposed that the hidden variables exist, and his inequalities are a statement of a set of requirements that must be met if the hidden variables are to be exclusively local (Bohm and Hiley, 1993, p. 143). Most crucially, “Bell’s theorem is not a property of quantum theory. It applies to any physical system with dichromatic variables, whose values are arbitrarily called 1 and -1 ” (Peres, 1995, p. 162).

Two remarkable results flow from the Bell inequalities. First, when applied to quantum mechanics, the wave function that describes the effect (while ignoring the causes) of the polarization of entangled photons violates the inequality. In other words, although the wave function ignores the cause of the entanglement, the violation of Bell’s inequality says that irrespective of the details of that causation, it must include non-local causal entailment. Second, and most crucially, “Bell’s inequality has been tested in a large number of experiments and generally speaking the inequality has been found to be violated” (Bohm and Hiley, 1993, p. 144). In other words, the observed violation of Bell’s inequality says that irrespective of the details of that causation, it must include non-local entailment.

The fact that both the wave function and the experiments violate Bell’s inequalities has provoked a most peculiar response in the physics community. Many physicists interpret the result as proof that “Bohr won; Einstein lost” (Jaynes, 1989). After all, the experiment agrees with the quantum mechanical description, “proving” that quantum mechanics is right, and that if there is a problem, it must be with relativity.

This misrepresents Einstein’s claim. Einstein never claimed that one could produce an effect that violates the quantum mechanical description. He did

claim that he could produce a result that quantum mechanics *ignores*. The experiment demonstrates that there exists a causal entailment structure, hidden from quantum mechanics, but entailing the effects described by quantum mechanics nevertheless. The hidden variables that Einstein affirmed and Bohr denied *are there*. Einstein's original claim that quantum mechanics is not complete is vindicated. For those keeping score, Einstein won; Bohr lost.

However, what Bell shows is not quite what Einstein expected. Einstein expected the hidden variables to be strictly local. Since Bell's inequalities are violated, some hidden variables *must* be non-local. According to Bohm, the non-locality reflects the influence of the whole upon the parts. Thus, reality has top-down as well as bottom-up causation. This is a paradigm shift that Einstein could not countenance; it is far more radical than the one that he made in order to generalize Newtonian mechanics into relativity.

Despite the fact that non-locality of the hidden variables did not turn out the way Einstein wished, something more important is revealed. Bohm and Hiley (1993, p. 353) see it pointing to a way of resolving the contradiction between relativity and quantum mechanics: "We need a new notion of order that will encompass these different kinds of unbroken wholeness, which could open the way for *new physical content* that includes relativity and quantum theory but has the possibility of going beyond both" (emphasis added).

Implicate Order

Bohm sees the influence of the unbroken whole of reality as being felt at every point in reality. He calls this enfoldment, and supposes that every point in reality contains some essence of the whole folded into it. "We may call this order implicit, but the basic root of the word implicit means 'enfolded'" (Bohm and Hiley, 1993, p. 354). In other words, the enfolded order can be inferred from direct experience, but is not itself directly experienced. As a result, he chooses *implicate order* as the term for the causal entailment structure by which the whole influences the properties of the parts.

Those parts also influence the whole. Bohm calls that entailment process unfoldment or explication. It is the opposite of implicate order, and he chooses *explicate order* as the term for the influence of the parts on the properties of a whole. He sees the two entailments operating in a complementary fashion: "Whatever persists with a constant form is sustained as the unfoldment of a recurrent and stable pattern which is constantly being renewed by enfoldment and dissolved by unfoldment. When the renewal ceases the form vanishes" (Bohm and Hiley, 1993, p. 357). The whole entails the parts and the parts entail the whole.

The Impredicative Character of Implicate Order

Curiously, Bohm overlooks the structural similarity between this interaction and impredicativity. "Impredicative" is mathematical term defined by Kleene (2000, p. 42): "When a set M and a particular object m , are so defined that on the one hand m is a member of M , and on the other hand the definition of m depends on M , we say that the procedure (or the definition of m , or the definition of M) is impredicative. Similarly, when a property P is possessed by an object m whose definition depends on P (here M is the set of the objects which possess the property P). An impredicative definition is circular, at least on its face, as what is defined participates in its own definition."

We can imagine the interaction of enfoldment and unfoldment as being analogous to the impredicative relationship. The distinguishing property of m is that it is a member of M ; this is enfoldment. The distinguishing property of M is that it contains m as a member; that is unfoldment. The relationship between M and m is defined by the inferential entailment that constrains their influence on each other. In impredicatives, as in Bohm's dynamic unfolding and enfoldment, it is not the objects but the relationship that matters.

An impredicative model of Bohm's orders would describe many of the properties that he attributes to them. In particular, the interaction of top-down and bottom-up entailment leads to an internal ambiguity fully in keeping with Bohm's expectation that the world is neither random nor unambiguously determined. Instead, Bohm and Hiley (1993, p. 324) say, ". . . our overall worldview is neither absolutely deterministic nor absolutely indeterministic. Rather it implies that these two extremes are abstractions which constitute different views or aspects of the overall set of appearances."

Holomovement Is Not a Hologram

Instead of impredicativity, Bohm chooses a far less useful analogy. He compares enfoldment to the construction of a hologram from a scene, and unfoldment to the recovery of the scene from the hologram. Since he expects the process to collapse if the dynamics ever halt, he calls the combined interactions of implicate and explicate order the *holomovement*.

It is unfortunate that in trying to describe the properties of these entailments, he compares them to a hologram. The analogy breaks down in at least two particulars. First, unlike Bohm's holomovement, a hologram is not a dynamic process; it is a static object. It is a recording of a three dimensional scene in exactly the same sense as a pattern of microscopic bumps on a compact disk is a static recording of a two dimensional scene. In both cases, to unaided human sensibilities the recording suggests nothing resembling the recorded scene. In both cases it requires special equipment to encode the

scene to the storage medium and other special equipment to decode the scene from the storage medium. However, the medium itself is static, and the encoding/decoding processes are strictly mechanistic.

It is crucially important to realize that *holography is a linear system* (Goodman, 1968, p. 203). Thus, it is not merely a mechanism, but rather it is among the simplest sort of mechanisms. Its operation is easily tractable by the principles of traditional physics, without the necessity to resort to relativity, quantum mechanics or chaos theory, much less any sort of new physics.

There is a great deal of confusion in the non-optical literature over the supposition that if the hologram is broken into pieces then each piece contains the entire scene. A typical example claims that "Unlike normal photographs, every small fragment of a piece of holographic film contains all of the information recorded in the whole" (Talbot, 1991, pp. 16–17). This is simply not the case. It is the case that if the hologram is constructed by the method of Leith and Upatneiks, then a piece of the hologram contains information that allows for the *partial* reconstruction of a two dimensional image of the original three dimensional scene *from a particular perspective* (Goodman, 1968, p. 220). This is a direct consequence of the ontologically reductionistic character of holography. The reason it works is that the three dimensional scene is fractioned into many two dimensional scenes each from a different perspective, and each disjoint fraction is concentrated in a different location on the plate. If and only if *the information from all those disjoint two dimensional perspectives is combined* is the original three dimensional scene recovered. The unbroken (and inherently unbreakable) wholeness of the holomovement is thus fundamentally different from the inherent disjointness of the parts of a hologram.

Like any other linear system, the holography is entailed exclusively by bottom-up causation. In this process the property of the whole is nothing but the sum of the properties of the parts. It is inexplicable that the hologram should be mistaken for a paradigm of the influence of the whole upon the parts. In holography, the parts of the scene disjointly entail the parts of the hologram, and the parts of the hologram disjointly entail the parts of the recovered scene.

There is a second and more significant particular in which the analogy between the hologram and the holomovement fails. There is not a hint of top-down causation or of any other non-mechanistic property in the operation of holography. A hologram lacks the key feature that accounts for the remarkable character of the holomovement and the new physics that it signifies; that key feature is its top-down causal entailment structure.

Wheeler: The Importance of Paradox

Wheeler goes beyond the idea that paradox is necessary for the discovery of new insights: "We need two paradoxes. Only then can we play one off against

the other to locate the new point" (Wheeler, 1980, p. 341). As it happens, two come to his mind. The first arises from relativity. If the universe starts with a big bang and ends in a big crunch, then the laws of physics have a finite lifetime. However, relativity is based on the presupposition that the laws of physics are forever immutable. The second paradox arises from quantum mechanics: the universe is both dependent on and independent of the act of observer-participancy.

In his thinking, the necessity for observer-participancy arises out of another principle that Wheeler attributes to Bohr. That is, "No elementary phenomenon is a phenomenon until it is a recorded phenomenon" (Wheeler, 1980, p. 356). In fact, this quotation appears over and over again in Wheeler's writings. Its meaning can be seen in the entangled photon problem. There, the phenomenon is the state of the polarization of the photon; that state, and its history, essentially spring into existence at the point that it is registered on the polarization detector.

To illustrate how observer-participancy might work, Wheeler recounts a game of "twenty questions" in which he was the questioner. Upon asking the first few questions, he received rapid answers, but the more questions he asked, the longer it took the participants to come up with simple yes-no answers. After not many questions, he asked, "Is the subject a cloud?" To this the audience unanimously answered, "Yes!" Then they revealed the joke: they had agreed in advance and unbeknownst to Wheeler that there was no subject. They would simply answer the next question in a manner that was consistent with all the answers that arose before. Nevertheless, within an amazingly short round, both Wheeler's questions and the audience's answers rapidly coalesced into a constraint such that a cloud was the only object that would fit. He argues that "phenomena" are formed in much the same participatory manner. He is at pains to point out that observer-participancy does not necessitate consciousness (Wheeler, 1980, p. 359).

The Impredicative Character of Observer Participancy

Wheeler describes the universe as a "self excited circuit." More specifically he says, "If the views we are exploring here are correct, one principle, observer-participancy suffices to build everything" (Wheeler, 1980, p. 359). Significantly, his version of new physics overturns *all* the presuppositions of the old physics.

This self-excited circuit is not merely a loop. Rather it is a loop of hierarchy of containment. He describes that hierarchy as follows: "To endlessness no alternative is evident but a loop, such as: physics gives rise to observer-participancy; observer-participancy gives rise to information; information gives rise to physics" (Wheeler, 1990, p. 8). In this instance, the hierarchical relation "gives rise to" is analogous to the notion of containment. He could just

as easily have said that physics contains observer-participancy contains information contains physics. Although he does not use the word, such a structure of entailment is the same as an impredicative construct in mathematics. Impredicativity has several properties that satisfy Wheeler's requirements. It entails its own entailment structure, and revises it as the context changes. Most crucially, its ambiguities provide the flexibility that Wheeler supposes to be randomness in the observer-participancy world.

Although Bohm's ontological quantum fields and Wheeler's epistemological observer-participancy registration appear utterly different from one another, they are strikingly similar in structure. Bohm says that implicate order entails explicate order entails implicate order. Wheeler (1990, p. 8) says, "Physics gives rise to observer-participancy; observer-participancy gives rise to information; information gives rise to physics." In both cases the entailment structure is entailed by an impredicative loop, and the impredicativity entails the bizarre behaviors in both paradigms. "Causation" is a word that Wheeler avoids. Nevertheless, the real world corresponding to Wheeler's epistemology is, like Bohm's, a world entailed by simultaneous bottom-up and top-down causation. There is nothing like this in any version of the old physics.

Schrödinger: What Is Life?

From yet another perspective, one of the earliest and perhaps the most compelling calls for new physics arose from no less a light than Schrödinger. What motivated his call was the question, "How can events *in space and time* which take place within the spatial boundary of a living organism be accounted for by physics and chemistry?" (Schrödinger, 1944/1992, p. 3). He saw this as leading to a significant problem: "The obvious inability of present-day physics and chemistry to account for such events is no reason at all for doubting that they can be accounted for by those sciences" (Schrödinger, 1944/1992, p. 4). It was clear to him that the old physics was not up to the task but a new physics might be.

Schrödinger observed a glaring difference in scale and asked why organisms are so much bigger (on the order of ten thousand to ten billion times longer) than atoms? His answer was that individual atoms in any physical medium make large oscillations compared to their size, and that a structure made up of a few atoms lacks the stability needed to support the degree of organization observed in an organism. In contrast, in a structure consisting of many atoms, the effect of the vibrations of individual atoms cancels out, causing large structures to have more overall stability than small ones. However, Schrödinger then notes that this point turns out to be comparatively insignificant.

Nevertheless, it provides crucial background for a question that does matter. Schrödinger noted that genes are on the order of a few hundred atomic

lengths. The size is “much too small to entail an orderly and lawful behavior” according to the laws of physics (p. 30). Nevertheless, genes are both “highly ordered” and entail a high degree of permanence on the phenotypic properties of an organism. Why does the high order of a small molecule arise, and how does it entail a highly stable phenotype? More perplexing, how do they achieve this feat when the number of atoms in the genes “represent only a very small fraction of the sum total in every cell?” (p. 77). Contemporary physics provides no answer. Even worse, the phenomenon appears to violate the laws of physics.

Schrödinger does not explicitly say “paradox,” but he has identified one. The idea that the gene itself possesses negligible energy, but nevertheless entails the massive energies of an organism to become organized in a form that depends on the form but not the energy of the gene. It is worth noting that Schrödinger is pointing at the same sort of phenomenon as Bohm’s active energy, in which the form of one process entails the properties of a much more energetic process.

Radical Consequences: Major Presuppositions Overtaken

Reasoning from quantum mechanical principles, Schrödinger developed a model of the entirely counter-intuitive properties that he said the genotypic aperiodic crystal must have. Although Schrödinger himself dismissed the description as “general,” it turned out to be specific enough to lead to the discovery of deoxyribonucleic acid within a decade of his publication of it. In his own mind, the model entailed “just one general conclusion,” which Schrödinger confesses, “was my only motive for writing this book.” That conclusion is “that living matter, while not eluding the ‘laws of physics’ as established up to date, is likely to involve ‘other laws of physics’ hitherto unknown” (1944/1992, pp. 67–68).

He sees a “striking contrast” between the order in mechanistic processes and the order in living processes. For example, in radioactive decay, there is no law of physics for identifying when a specific atom will disintegrate (p. 78). The lifetime of individual seemingly-identical atoms varies wildly. In contrast to the wild variations in the behavior of individual atoms, Schrödinger characterized the order entailed by the genome to present a situation that is unprecedented, and “unknown anywhere else except in living matter” (p. 79). In “striking contrast,” in an organism “a single group of atoms existing only in one copy produces orderly events, marvelously tuned in with each other and with the environment according to the most subtle laws” (p. 79) The context dependency of events observed in organisms directly violates the first presupposition of traditional physics that the behavior of the whole is the sum of the behaviors of its parts and nothing more.

In pondering the question of how an organism might maintain its order, Schrödinger noted that “we witness an event that existing order displays a power of maintaining itself and of producing orderly events” (p. 77). He explains that

he gets the idea from social organization rather than traditional physics. He is speaking of top-down causation. In so doing, he is suggesting a genuinely radical idea, that the new physics might overturn the presupposition that permeates conventional physics, namely that all events in reality are exclusively the effect of bottom-up causation (presuming they are caused at all).

The Impredicative Character of Super Physics

Remarkably, four decades before the hyperset theorists proved the logical coherence of impredicativity, Schrödinger anticipated that top-down causation might arise, from an impredicative entailment structure. "It might seem that something like a vicious circle is implied" (1944/1992, p. 77). Russell equated the vicious circle with impredicativity, a fact probably well known to Schrödinger. Thus, although the paradigms of Bohm, Wheeler, and Schrödinger are all underpinned by a structure that looks like impredicativity, only Schrödinger comes close to actually using the word.

Schrödinger considered living processes to be "too involved to be fully accessible to mathematics" (p. 3). Clearly, he considered organisms to be material processes within the scope of rational understanding that would be afforded by the new physics. Just as clearly, the new physics that he envisioned must do away with the presupposition that all the available information about a material process can be described by a differential equation whose solution is a map, $\sigma: \mathbb{R}^m \rightarrow \mathbb{R}^n$.

This paper has previously listed five presuppositions that are shared (or ignored, but not overturned) by all the currently accepted theories of physics. It is remarkable that by the observation of "fundamental differences in structure" in organisms in "striking contrast" to that of mechanisms, Schrödinger explicitly overturned three of them. By noting the context dependency of organismic behavior, he overturned ontological reductionism. By noting the impredicative character of the organism's imposition on the order of its parts, he overturned the presupposition of exclusively bottom-up causation. By noting that the description of living processes is "too involved to be fully accessible to mathematics" he overturned the presupposition that everything worth knowing is fully expressible in numbers.

Schrödinger does not discard the old physics. He sees it as a degenerate case of the new physics. Nevertheless, by explicitly overturning three of the five fundamental presuppositions of the old physics, the revolution in physics that Schrödinger proposed is far more sweeping in its scope than the paradigm shifts that followed either relativity or quantum mechanics. Given the genuinely breathtaking scope of his proposal, it is little wonder that speaking of the process of living he said "we must be prepared to find a new type of physical law prevailing in it. Or are we to term it a non-physical, not to say a super-physical, law?" (Schrödinger, 1944/1992, p. 80).

Conclusion

From three different perspectives Bohm, Wheeler, and Schrödinger, three of the most seminal thinkers in the history of physics, all make the same point. We need a superphysics, a coherent physical law in which the descriptions of effects currently explained by relativity, quantum mechanics and classical physics all fall out as degenerate cases. The reason they say it is needed is that contemporary theories of physics are incoherent. Relativity and quantum mechanics contradict each other, and the observed structural relationships in living processes violate the laws of contemporary physics.

The specific laws that constitute the new physics remain to be discovered. As with prior paradigm shifts in physics, resolution of the paradoxes and discovery of a coherent physics will require that some of the most cherished presuppositions of the old physics be overturned. Based on the discoveries of Jaynes, Bohm, Wheeler and Schrödinger it is reasonable to anticipate that the laws of new physics will be consistent with presuppositions akin to those in the following list.

1. The Gestalt is an understatement; the behavior of the whole is greater than the direct product of the parts (Dress, 2000).
2. Reality is characterized by inherently dynamic relationships rather than a progressive ratcheting through a sequence of quasi-static states.
3. Causation is ambiguous; more than one outcome might be coherent with a given entailment structure in a given context.
4. Events in reality are the effect of both bottom-up (the influence of the parts on the whole) and top-down (the influence of the whole on the parts) causation.
5. There is, neither in practice nor in principle, a complete or largest model isomorphic to reality.
6. To avoid paradoxes, reality, our partial models of reality, and the relationship between reality and our models of it must be considered.
7. Although there is no largest model, we can construct models such that we can gain insight about the process being modeled by asking questions about the model; we expect many of those models to be category-theoretic maps of the form, $\sigma: Y \rightarrow Y$, where Y is the set of impredicative maps. (Differential equations are a degenerate instance of this more general map.)

The claim of a need for a new physics, based on presuppositions similar to the above, arises from within the physics community. It is not an instance of life scientists trying to tell physicists how to do their job. In fact, there is a lesson here for life scientists. Despite the fact that the processes of life and mind unfold on a physical substrate, and must be consistent with the laws of physics,

there is no need to try to shoehorn theories of life and mind into the narrow constraints of the old physics. The physics community is beginning the process of replacing the old physics with a radical new physics, and it affords a much richer milieu in which to understand the processes of life and mind.

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