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Causation and Corresponding Correlations

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Experior Assessments

Corresponding correlations is a method that allows us to infer formal causation from correlational data. In this paper, causal terms are traced to their philosophical and etymological roots. It is argued that causes are parts of their mutual whole (effect). Nominalism, normal distributions and disjunctive causes are linked. Causal manifolds and sampling by potential are used to model conjunctive causes. Corresponding correlations are then demonstrated through simulations, in which causal relations are differentiated from spurious correlations. An algebraic method for unraveling confounded variables is presented. Distinctions between laws and causes are made and related to corresponding correlations. The conclusion is that corresponding correlations should be a significant advance in causal inference.

This paper is an introduction to corresponding correlations and demonstrates the principles that undergird corresponding regressions. The first part of the paper addresses issues in the philosophy of science that have diverted scientific attention away from the possibility of causal inference from correlations. The second part explains how such inference is possible. Corresponding regressions (Chambers, 1991) is a method by which the asymmetrical logical relations between causes and effects are inferred without recourse to experimental manipulation, quasi-experimentation or to traditional structural equation modeling. Corresponding correlations and corresponding regressions are framed in classical ontological terms that afford precise epistemological and arithmetic definition for causal inference. Whereas most statisticians and researchers assume the inference of asymmetrical functional relations is impossible without experimentation, corresponding correlations and corresponding regressions demonstrate that this forbidden inference is possible, given proper conceptualization, sampling and calculation.

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The mathematics of corresponding regressions/correlations is deceptively simple — a characteristic that may slow its acceptance in the development of ideas, although most researchers admit that the inference of causation from correlations would be a great advance in statistics. Many assume such inference to be categorically impossible while a few researchers hope that an extremely complicated equation may yet be developed that solves the problem. The structural equations discipline (Loehlin, 1987) seems to hope for a complex equation and has developed a taste for arcane mathematical procedures, producing a plethora of methods, with none standing clearly above the rest, and none allowing inference of causation without experimental manipulation. Reaction to corresponding regressions/correlations can be summarized as "The math can not be that simple!" Although the equations may actually be simple, the logic required to understand the method is complicated.[1]

Logic and Experiment in Causal Inference

The logical asymmetry of causation rests in the "part to whole" relationships existing between independent (IV) and dependent variables (DV). As parts, independent variables are structurally simple (L. *sine plex*, without doubling or folding). Effects, on the other hand, are complex wholes (L. *con plecto*, to double, to weave, to fold). By composition (L. *con positus*, to set together), processes comparable to arithmetic operations (+, – , * , /) weave parts into complex dependent variables. These combinations (L. *com binus*, to double) produce wholes that are dependent on their parts (L. *pars*, to separate). The parts are operationally (L. *operor*, to make, do or form) composed into wholes, by rules of logic and principles of form.

Aristotle (see Rychlak, 1981) referred to four types of causes: the material, formal, efficient, and final. Each of these four causes contributes to the constitution of substances. The *material cause* concerns the undifferentiated stuff of which substances are composed. Lacking concrete existence on its own, *materia prima* nonetheless has infinite potential. It can acquire an infinite variety of forms. *Formal causes* are the attributes that define (L. *de finio*, to limit) the infinite potential of prime material. These definitions include shapes, qualities, operations and other abstract forms. *Substances* are said to be *hylemorphic* (G. *hyl morphe*, combined form), that is, they are composed of both material and formal causes

Hylemorphism implies the combination of matter and form through operations that constitute asymmetrical orders of inclusion. Being hylemorphic, the constitution of substance includes both form and material. However,

[1]The 1997–98 archives of the internet group SEMNET contains an extensive debate on corresponding regressions. Access the archives at http:bama.ua.edu/archives/semnet.html

pure material (*materia prima*) and pure form (numbers, angels), on their own, do not imply substance. Matter and form have their own being independent of substantive existence. Consequently, hylemorphic wholes (substances) include parts but these parts do not include the substantive wholes that they constitute. Such constitution by inclusion is illustrated by the classical syllogism.

The syllogism (G. *syn logizesthai*, together reason) is a logical form that expresses necessary conclusions that follow, by definition, from several propositions (L. *proposito*, to put or to throw forward). The syllogism consists of three parts, the major premise (All men are mortal), the middle term (Socrates is a man), and the conclusion (Socrates is a mortal). In this syllogism, Socrates logically implies man, which in turn implies mortal. Socrates is the more complex construct, while man and mortal are respectively more simple and universal in their application. Socrates is tautological with man and mortal, in that they are parts of Socrates by definition. But there is an asymmetry in this order of inclusion. Although both mortal and man constitute Socrates, neither of them logically implies Socrates, since some mortal men are not Socrates. On the other hand, the substantive Socrates does imply both man and mortal. The syllogistic asymmetry thus hinges on one-way logical implications.

Historically, measurement of constitutional asymmetries has been problematic because syllogistic definitions can become fanciful. To say "All men are donkeys. Socrates is a man. Therefore Socrates is a donkey" is a formally valid but substantively fanciful syllogism. It is based on premises that are stipulated without material grounding. In science, there must be something to link the premises of syllogisms to observable facts. For the last few centuries, most people have believed that only experimentation can make this link.

Francis Bacon (1620/1989) championed the cause of experiments and induction in his sixteenth century book *Novum Organum*. He insisted that premises must be more than the mere opinions of individuals or groups. The root of the word stipulate (L. *stipes*, crowd) concerns propositions made by a collection of people. In Bacon's time the stipulating crowd consisted of the church fathers, who taught Thomas Aquinas' revision of Aristotle (see Russell, 1972). This revision occasionally stretched the limits of sensibility, for example, by challenging even the notion of hylemorphism, through declaring that angels exist without substance. These and more mundane propositions led Bacon to catalogue sociological and psychological factors that mislead scientific inquiry. He referred to these as the Idols of the Tribe (species-wide illusions), Den (personal misconstructions), Market Place (fallacious social attributions), and Theatre (theoretical errors). Bacon argued that there must be a tractable correspondence between ideas and events and he suggested that experiments and induction, rather than deductive logic, were the best means of drawing these connections. Of course, history has shown that experiments do help define the asymmetries in experience. Experiments tell us that manipulating a cause changes an effect while manipulating an effect does not change a cause. Experiments thus reveal asymmetries that may, in turn, support the syllogisms of scientific theories. The use of experiments, however, de-emphasizes formal causation while tipping the hylemorphic balance over to material causes.

As appealing as Bacon's philosophy of experimentation was to physical scientists, astronomers faced seemingly insurmountable challenges in inferring causation. Experiments on heavenly bodies were impossible although systematic measurements of the heavens had been made since before recorded history. Whereas these observations had allowed for predictions of even rare events, such as solar eclipses and comets, the understanding of the heavens was still very much an abstract logical endeavor. The Copernican theory challenged Ptolemy's views through an intellectual reconstruction of ancient observations (see Russell, 1972) instead of by an experiment. There were no physical manipulations, only a more creative intellectual vision and more consistent mathematical models of the heavens. Consequently, it was to mathematics and logic that the non-experimental sciences turned to provide structure to their observations. This movement toward abstract models reached a high point with Newton's theory of gravity, which was made possible by the newly invented discipline of calculus.

With calculus came a language of functions and later, of correlations, that helped track changes in place and the variations of form. Functions, however, failed to disclose asymmetrical relations (Bunge, 1979). Even with functions, the stipulation of "what was a function of what" was still a matter of speculation, not of inductive certainty. Functions still required the genius of a Newton and without such intelligence, function models were likely to be misdirected by the Idols of the Tribe, Den, Market Place and Theatre. As Bacon (1620/1989, p. 149) suggested "It is only for God, and perhaps for angels or intelligence at once to recognize forms affirmatively at first glance of contemplation." Although near-angelic intelligences, such as those of Newton, were very rare, Newton's example did spawn refinements of the mathematics of functions and probabilities by such men as Bayes, LaPlace, and Gauss (see Stigler, 1986). These refinements, however, were insufficient to replace the subtle contemplation of genius. The mathematics of Gauss and others could not inductively reflect asymmetrical functions "at first glance."

Perhaps frustrated by their inability to infer asymmetrical forms, Gauss and others turned their attentions to developing a theory of measurement error. If they could not directly contemplate the forms of angels and abstract causes, they would at least facilitate a consensus on what they could measure. Gauss studied the characteristics of mistaken observations, as might happen, for example, when two astronomers disagree on the location of a particular star at a certain time.[2] It was soon discovered that measurement errors tended to be normally distributed and that these "personal equations" had systematic effects on the correlations and functions that were used to model phenomena. Correlations were subsequently expected to serve as indices of both reliability and validity, as detailed in the classical theory of true scores (Nunnally, 1978). Interpreting the meaning of functions and correlations soon became very complex since the assumptions of normal measurement errors were confounded with the mechanics of uniform functions. This confound is most clearly expressed in modern structural equations modeling.

In their primer for structural equations modeling, James, Mulaik, and Brett (1982) suggest that asymmetrical functional relations are essential for causal inference. They also admit that there are no received non-experimental statistical methods for determining asymmetrical relations. Structural equations are still just systems of functions stipulated a priori as theoretical propositions or opinions. The advances provided by structural equations are largely refinements in analyzing systems of functions that tap latent variables and in procedures for analytically decomposing correlation matrices, according to theoretical models. But the roots of these refinements are found in classical true score theory and multiple regression analysis, which, in turn, are traced back to Gauss.

It is not surprising that working from essentially the same assumptions as Gauss, structural equation researchers were greatly concerned with the effects of measurement error on theoretical models. It is as though researchers from both eras said, "It is true that we can not determine causal structures even with perfect observation but we can at least patch up nonsensical inconsistencies in what we can observe." This reaction essentially changed the subject from "What is the valid causal model?" to "Can we all agree on what probably exists on a concrete level, below the insights of angels and genius?" As with the philosophers of previous centuries, the mathematicians of measurement error, from Gauss up to Joreskog and Sorbom (1984), implicitly focused their frustrations around the question of "What is nonsense?" A review of even earlier debates will help clarify the ancient and modern entanglements of this thorny question.

Although questions of "How many angels can stand on the point of a needle?" can lead to refined mathematical notions, such as those of Cantor sets and nested infinite series (Russell, 1993), there is much room along the way for nonsense. The Inquisition and its peculiar studies of fallen angels

[2]Boring (1950, p. 134) tells the story of the Greenwich astronomer Maskelyn, who fired his assistant, Kinnebrook, for observing stellar transits almost a second later than Maskelyn.

provided many examples of such nonsense. Fear of colluding in or running afoul of nonsense kept many mathematicians from even contemplating the non-experimental perception of forms. They chose to model more concrete forms, thus avoiding the punishments suffered by Bruno and Galileo. Eventually, however, ridicule of the idea of immaterial forms (angels) served as a common sport for those whose interests were worldly and driven by the senses. "Just how much sense can form make anyway, compared to material reality? We can all agree a lump of gold is gold but we will disagree about angels and other abstract forms forever."

In the Middle Ages (see Robinson, 1981), questions concerning the reality of pure form led to a great debate between the nominalists (who believed universal predicates were merely names) and the realists (who believed abstract universals were real in some sense, as were angels). The nominalists did not believe in the literal reality of the properties and operations that make up formal causes, any more than they believed in angels. These properties seemed to fall outside the purview of common sense. No mathematics existed to demonstrate such inclusive relations inductively and no experiments could grasp the subtle natures of formal causes. With the gradual waning of church authority in the Middle Ages, universal abstractions seemed ephemeral and immaterial. Forms, such as taxonomic hierarchies had been handed down by authority figures and could not satisfy the rising force of liberal individualism and commerce.[3] As the authority of the church diminished, alternative theories of causation arose. In the fourteenth century, William of Ockham translated his liberal politics into an epistemology that emphasized attention to particular effects instead of contemplation of general (catholic) abstract causes. He went on to challenge the notion of abstract causes as in the statement below.

> There is no unitary, unvaried or simple thing in a multiplicity of singular things nor in any kind of created individuals, together and at the same time. If such a thing were allowed, it would be numerically one; therefore, it would not be in many singular objects nor would it be of their essence. (ca. 1340/1955, pp. 873–874)

By these words, Ockham denied the authority of major premises, such as "All men are mortal," and shifted the focus away from catholic forms onto concrete individual objects. The result was a constriction of the range of legitimate topics of scientific discourse, disguised as an appeal to parsimony. The nominalists simply defined away the authority of abstractions by letting

[3]For example, in the *Divine Comedy*, Dante provides a model of the universe for classifying men and angels.

nature's individual and concrete objects stipulate the focus of scientific inquiry. Perception in the form of measurement/sampling would cast the premises of syllogisms while manipulation would define the order of causal inclusion. Ockham's Razor had made a deep cut for the sake of parsimony and concretism. Francis Bacon expanded this outlook.

> The syllogism consists of propositions, propositions of words; words are the signs of notions. If therefore, the notions (which form the basis of the whole) be confused and carelessly abstracted from things, there is no solidity in the superstructure. Our only hope, then, is in genuine induction. (1620/1989, p. 107)

Bacon argued for generalizing from particulars (induction) as a means of avoiding deduction based on vacuous (authoritative) premises. But induction on its own, without the guidance of angels or genius, tends to be haphazard and full of measurement errors. Bacon knew this but believed that the experimental manipulation of objects would focus the scientist's mind. This experimental focus would protect against the Idols of the Tribe (haphazard perception) and against the Idols of the Theatre (erroneous theory). Reliable experimental manipulations were Bacon's way of grounding premises in reality. He traded subtle abstract forms (logic) for reliable actions (experiments).

The nominalist attraction to the concrete and particular, transformed scientific perception into a type of statistical sampling that severely limited generalization. Following Ockham's direction, the nominalists' sampling focused on particulars, such as the individual man Socrates, as he stands before us, rather than studying the propagation of universal predicates, such as the classes of man and mortal, across groups of individuals. Behaviorists (Johnston and Pennypacker, 1980) carry this philosophy to its extreme and question the use of any cross-sectional design. They advocate n=1 studies, claiming we only learn about organisms by studying the changing properties of individuals. Cattell and Cross (1952) showed, however, that the same personality factors emerge from both cross-sectional studies (R-technique), based on groups, and from n of 1 (P-technique) studies, based on individuals across time. Thus, neither sampling strategy is without merit when nature presents us with hylemorphic wholes, such as the man Socrates or with larger samples of such whole individuals. The nature of all men tends to be reflected in each individual man. A problem occurs, however, when that which our sample throws before us is not a coherent whole but a collection of disparate sensations. Within the chaotic flux of the senses, we tend to become preoccupied with measurement errors as a means of obtaining consensus on what we can see. Such consensus seeking, however, predisposes us to the Idols of the Market Place, as evidenced by studies of attribution and influence (see Chialdini, 1984). The upshot is that the search for wholes is particularly problematic for those who would infer causation without manipulation, since there are no apparent criteria for determining part/whole relationships in the haphazard flow of events.

The search for particular and concrete objects leads to a certain illusion of wholeness. What immediately strikes the senses appears to be "the whole story." Most advocates of structural equation modeling sample by collections that nature throws before them. By this haphazard process, nature and personal whim too often define the propositions that predicate the inquiry. Piaget's (1972) theory of intellectual development addresses the effects of such syncretic perception. The child's attention is fixated by irrelevant features, akin to Bacon's Idols. Piaget (see Flavell, 1963) found that maturation beyond such random and egocentric fixations of attention only occurs with the mastery of abstract logic that allows for the systematic sampling of events. Lattice structures direct the mature child's attention to all possible forms, expanding the perspective beyond the haphazard and material, toward the hylemorphic nature of substances. We shall return to this theme later, in discussing sampling by potential.

In order to bring some stability to their observations, structural equation researchers use rules of thumb, such as the assumption that where there is correlation, causation is at least plausible. Other guidelines include archaic statistical notions, such as the belief that normal distributions imply wholes. It is well understood, however, that every correlation matrix can support the plausibility of a near infinite number of causal models. This is not much to grasp in the chaos of haphazard experience. Furthermore, the belief that normal distributions indicate natural wholes was discredited soon after Quetelet's (see Stigler, 1986) initial enthusiasm for normal distributions. The result is that structural equation modeling is an unstable endeavor, having as its major appeal the investigator's freedom to theorize at will, while maintaining an aura of mathematical sophistication.

It is a curious twist of reason that haphazard sampling has led some researchers to the erroneous belief that any systematic sampling is necessarily experimental. This mistaken belief goes further to claim that since structural equations are non-experimental they must not be based on systematic sampling designs. In other words, structural equations are natural and unplanned; experiments are systematic and contrived. This argument has been used against corresponding regressions, which stresses systematic sampling but without experimentation. Opponents claim that corresponding regressions is a type of experimentation, even if no physical causes are manipulated. In fact, however, sampling may be systematic and still not experimental, since we may logically sample from nature without actively manipulating causes. An experiment requires manipulation of a cause to bring about changes as an effect. Systematic sampling is an altogether different type of process. We may not be able to experimentally manipulate subtle abstractions but we can sample such variables systematically, looking for all the possible combinations of the causes in order to map the full phenomenon. Piaget (see Flavell, 1963) linked such systematic construction to formal operations. It will be shown that sampling by formal operations is especially promising because it makes possible the inference of causation without manipulation.

Those who advocate haphazard sampling procedures tend to let the circumstances of sampling stipulate their theoretical premises. Such induction is at best naïve. Causal functions are not essentially distributions. They may be instantiated but the resulting incidents are accidental in that very different distributions may be thrown by the same causal function. The instantiation depends on what causal values are passed to the function. Few if any structural equation samples uniformly cover the whole abstract range of variables. But even an extremely rare incidence, for example, the pairing of extremes from two otherwise normally distributed variables, point to latent underlying uniform causes.

To the realist, thrown distributions were merely the mixed up shadows of a haphazard collection of different variables, as illustrated in Plato's analogy of the Cave. Thrown distributions were viewed as accidental (L. *accidens*, falling from). The realists knew that, like shadows on the wall of a cave, instantiations may be only confounded variables, spuriously thrown together but appearing as coherent material wholes to the senses. Such variables may, in fact, have identities of their own, separate from the illusory material wholes of coincidence and perception. Color, mass, electric charge, etc., exist at some level of abstraction, independent of their instantiations in particular objects. Because of this, the realist could conceive of causal relations as stretching across a range of abstract possibilities, rather than simply across some haphazard material ("natural") sample. This gave the realist the edge in discovery because he or she could use combinatorial analysis to create and isolate causes and effects that did not yet materially exist in any sample. Thus by the power of intellect, the realist could see beyond the information given (see Bruner [1973] for an in depth treatment of the child's going beyond the information given).

Most structural equation researchers do not conceptualize substances as hylemorphic. They ignore the manifold definitions of material that are provided by form. The "merely possible" sounds too much like a justification for unobserved angels standing on the point of a needle, threatening a regression to the Bonaventurian belief in spiritual matter and transubstantiation (Brugger and Baker, 1972). Atomism and concretism, instead, seem to be no-nonsense alternatives. But there are non-spiritual expressions of hylemorphism. Structural equation researchers forget the industry of biologists and botanists searching the globe to fill in missing evolutionary possibilities in collections of specimens. Instead, most structural equation researchers place their trust in the auguries that nature throws haphazardly as material samples.

Some structural equations users may respond at this juncture that sampling need not be haphazard and that some (quasi-experimental) measures reveal asymmetrical relations independent of formal logical analysis. Time is most likely to be the champion of this argument:

> Causes come before effects. Effects never follow causes. Time at least narrows the field of possibilities. The sophisticated mathematics of fit indices can shore up the rest.

This argument is wrong because formal asymmetries may exist even if part/whole compositions occur simultaneously or have always existed. Because it takes no time for 2 to equal 1+1, time is not of the essence of formal asymmetrical relations. Definition by inclusion, however, is fundamental to all part/whole relationships. The primacy of form over change does not mean temporal sequences do not exist. Aristotle defined *efficient cause* as the process of becoming from one form to a contrary form across time. But as substances change across time, even the changes have form. When a carpenter assembles a house he or she engages in an efficient cause transformation across time. At every point in the sequence, the production has form and substance, making form more fundamental than time and change. Efficient cause may be the foundation of experimentation but formal cause is the foundation of efficient cause. This underscores the primacy of logic over experimentation.

In summary, science grew up from Aristotle's attempts to systematize both logic and the classification of natural objects. Followers of this approach could use deductive logic, based on abstract premises (taxonomies), handed down by authorities. With the unrest that followed political and economic liberalization of the church state, nominalists turned to induction and generalizations derived from material particulars. The experiment eventually proved to be a highly productive means of inferring causation. Unfortunately, those disciplines that could not manipulate objects could not use experiments to determine asymmetrical relationships. The problem continues today. Structural equation researchers attempt to bypass experimentation by using correlations to choose between competing deductive causal models. But there will always be more than one plausible model supported by the data, since traditional correlations are symmetrical. Corresponding regressions/correlations provides a third path to causal inference, one that avoids the ambiguities of correlations and the concretistic constraints of nominalism, while allowing a hylemorphic model of causation.

Causal Manifolds and Sampling by Potential

Formal causation concerns the definition of a substance or process by the combination of independent components: features, qualities, properties, etc. The systematic perception of component parts requires an algorithmic elaboration of nature, one that Chambers (in press) [with due reference to Piaget and Bruner] called sampling by potential. In this sampling strategy the investigator looks beyond the immediate material sample, into all the logical possibilities. This strategy is comparable to what the biologist does when venturing far afield to discover an unknown but logically possible species. Such research requires greater industry in sampling than is common in most behavioral sciences but it promises a fuller view of the causal possibilities.

Sampling by potential is primarily concerned with causes as arithmetic sequence functions (see Wells and Tilson, 1997, p. 87). In arithmetic sequences, the first differences between consecutive terms are constant. This allows for operations that are invariant to additive constants. The difference x1–x2 is the same as the difference (a+x1)–(a+x2). Consequently, linear equations maintain across the entire ranges of their variables, since we can generate a range by simply adding progressively greater constants to consecutive variables. We generate a uniform linear range by multiplying n by a, in the equation (an+x1)–(an+x2). The difference will remain the same across the ranges so long as a and n are the same values for both x1 and x2. This sameness allows us to generate wide, uniform ranges of x1 and x2.

The constancy of first differences characterizes causal relationships that are said to be conjunctive (Bunge, 1979). Conjunctive causes concern arithmetic operations that generalize across the entire ranges of the x variables. To get equivalent first differences, however, it must be possible to keep the constants a and n equivalent for both (an+x1) and (an+x2). This does not happen with normally distributed causes, since there are very few if any incidents in which extreme values of x1 are combined with extreme values of x2. The reason for this is that extreme values of either x are very rare, even when matched with average values of the other x. Their combined occurrence (equivalent values for a and n for both x1 and x2) are so rare that inductive samples fail to reflect the property of the equivalence of first differences. The result is the equation y=x1+x2 rarely or never maintains for extreme values of x1 and x2 because these extremes are rarely or never combined in the cause. Thus, in effect, normally generated causes only allow disjunctive causal relationships, in which extremes of y tend to be largely due to either an extreme x1 or an extreme x2 but not to extremes of both x1 and x2.

Using uniformly generated values of x1 and x2 allows a conjunctive causal relation, whereas normally distributed values force a disjunctive relation.

This is a problem found throughout statistics. Indeed, most of structural equations modeling is based on the assumption that all variables should be normally distributed, thus implying all causes are sampled as disjunctive and not instantiated as linear sequences. The preference for normally distributed variables goes back to Gauss and models of normally distributed random errors. To correct for this disjunctive problem, users of structural equations should collect uniform samples of putative causes. Such uniform samples are rarely collected, however, because researchers do not know which variables are causes and which are effects. Furthermore, the standard probability tests are designed for normally distributed variables. As a result, the failure to adequately sample causes obscures basic properties of linearity in the data. Because of the way corresponding correlations works, however, we do not ignore the distinction between conjunctive and disjunctive causes.

Table 1 simulates a causal process based on linear sequences. It was generated by the additive *conjugation* (combination in all possible ways) of two independent variables (x1 and x2), each having eight levels. The additive conjugation produces the manifold of all possible joint instantiations of the independent variables, allowing for conjunctive causes and the first difference property. This conjugation should not be viewed as part of the corresponding correlations analysis but merely a simulation of conjunctive causation. If the investigator samples uniformly across uncorrelated causes, the sample space will tend to recover nature's latent manifold combinations, making corresponding correlations analysis possible by induction.

The manifold is a map of the products of operations that are applied across the whole logical ranges of the causes. Due to the central limit effect, the dependent variable (DV) lacks the uniformity of the IVs and tends toward a triangular distribution. This is because there are more ways to generate mid versus extreme ranges of dependent values. For example, if our causes are based on five-point scales (allowing the effect to range from 2 to 10), we can compose the midrange effect "6" in several ways: (6=5+1=3+3=4+2 etc). The extreme effect "10," however, is only generated by (10=5+5).

Some researchers have suggested that the mere presence of uniform and triangular (normal) correlates is sufficient for causal inference but this is not so. The normal variable may be only spuriously correlated with the uniform variable. Their correlation may be random or incidental to a mutual common source in a third variable. The differences in their distributions may also be due to unrelated environmental constraints. We must look deeper than just the given material distribution to see the logic of the causal connections.[4]

[4]The assumption that IVs are potentially uniformly instantiated is consistent with the practice of using equal cell sizes across the factors of an ANOVA.

Table 1

8*8 Addition Manifold

Parts + Parts=Wholes			
x1+x2=y	x1+x2=y	x1+x2=y	x1+x2=y
1+1=2	2+1=3	3+1=4	4+1=5
1+2=3	2+2=4	3+2=5	4+2=6
1+3=4	2+3=5	3+3=6	4+3=7
1+4=5	2+4=6	3+4=7	4+4=8
1+5=6	2+5=7	3+5=8	4+5=9
1+6=7	2+6=8	3+6=9	4+6=10
1+7=8	2+7=9	3+7=10	4+7=11
1+8=9	2+8=10	3+8=11	4+8=12
5+1=6	6+1=7	7+1=8	8+1=9
5+2=7	6+2=8	7+2=9	8+2=10
5+3=8	6+3=9	7+3=10	8+3=11
5+4=9	6+4=10	7+4=11	8+4=12
5+5=10	6+5=11	7+5=12	8+5=13
5+6=11	6+6=12	7+6=13	8+6=14
5+7=12	6+7=13	7+7=14	8+7=15
5+8=13	6+8=14	7+8=15	8+8=16

The linear independence of variables x1 and x2 is reflected in their zero correlation, even though they are exhaustively combined (conjugated) in all 64 possible ways. Correlations between IVs indicate confounded constructs and should be perceived as measurement or sampling problems. If two constructs truly are independent, they should be capable of complete conjugation, at least in principle. When correlations between causes exist, it is probable that disparate variables have been confounded by name, measurement, sample and/or by nature. An example of the latter would occur when a sour taste naturally accompanies a green color, then the taste and color are aspects of a common underlying variable/process and can not be conjugated as they are. They are not really independent parts but rather aspects of a common latent process. Such confounding does place a limit on the usefulness of corresponding correlations (which assumes conjugation of independent variables).

Artificial means, such as genetic alteration or the dyeing of apples, could be used to expand the possibilities of the objects. These kinds of dialectical strategies have, in fact, been argued as central to scientific creativity (see Rychlak, 1988). If creative steps are impossible, however, the investigator can simply explicitly clump green and sour into a single construct, to be conjugated, as a unit, with other independent variables. This is what factor analysis does when used to extract variables with simple structure. Such latent orthogonal variables are optimal for corresponding correlations. This topic is discussed in more detail in Chambers (in press).

Corresponding Correlations

If nature produced its composites by only one operation, say addition, and if our measures were true to nature's scales, then causal inferences would be as easy as determining which variable is the sum of which others. But insofar as nature uses all four operations and because our measures are often not true to scale, we can not read nature's compositions so easily. We need means of assessing the presence of composite wholes that arise from any operation and from any scale of measurement.

The essence of corresponding correlations is that parts are correlated one way in the extremes (upper and lower quartiles) of their composite and the opposite way in the midrange of their composite. In the causal model y=x1+x2 the values of x1 and x2 will be positively correlated toward the extreme ends of y. High x1 plus high x2 equals high y (10=5+5). Low x1 plus low x2 produces low y (2=1+1). In the midrange of y, on the other hand, we find those opposite values of x1 and x2 canceling one another toward the middle of y (6=5+1). Thus x1 and x2 are correlated negatively in the midrange of y but positively in the extremes of y. This polarization of correlations between x1 and x2 does not occur when we compare x1 and x2 across the ranges of either x1 or x2.

Table 2 displays the squared correlations, retaining signs, between x1 and x2 across the ranges of y, when y=x1+x2. Squared correlations are used because they can be summed with other squared correlations. Squared correlations form a linear scale, unsquared correlations do not. In Table 2, four ranges and four samples are defined for each causal model. The extremes of y were those values of x1, x2 and y that fell into the upper and lower quartiles of y, after sorting all the data by y. Any "tied" values at the boundaries of the quartiles were randomly distributed by the computer. The midrange of y was created from the remaining values corresponding to the inner quartiles of y. The extremes of x1 were those values of x1 and x2 corresponding to the upper and lower quartiles of x1, after sorting by x1. Midrange x1 included those values that correspond to the inner quartiles of x1. The samples were created from manifolds, uniformly distributed random numbers and normal random numbers based on n=100 per simulation.

Polarity scores were formed by multiplying the squared correlations (retaining signs) between x1 and x2 in the midrange of y with the squared correlations from the extremes of y.[5] For example, the polarity of the 4*4 manifold is –.15=.26* –.59.

Table 2

Corresponding (Squared) Correlations Between x1 and x2 Retaining Signs

	4*4 Manifold	8*8 Manifold	Average for 200 Replications: Uniform Random	Average for 200 Replications: Normal Random
Extreme of y	.26	.34	.33	.12
Mid-range of y	–.59	–.65	–.63	–.53
Polarity	–.15	–.22	–.20	–.06
Extreme of x1	.00	.00	.00	.00
Extreme of x2	.00	.00	.00	.00
Polarity	.00	.00	.00	.00

Polarity scores for all samples indicate the causal directions but the score (–.06) for the normally distributed variables is highly attenuated. This three-fold reduction in power demonstrates the impact of disjunctive causes. If the extreme values are not adequately represented then the resulting disjunctive causes are missed by corresponding correlations. Such causes implicitly violate the first difference property of linear sequences, since the constants (a and n) defining the scale in the equation (an + x1)–(an + x2) are rarely if ever the same for both x1 and x2 with normal generations.

As expected, the polarities based on the uniform causes do allow us to infer causation. The polarities created by sorting by the independent variables, on the other hand, are zero. No polarization occurs across the ranges of x1 (or x2). From these data we can determine that y is a composite of x1 and x2. These facts contradict the long held assumption that causation can not be inferred from correlations.

[5]In the subtractive model the correlation in the extremes is negative while that in the midrange is positive. But the polarization still occurs (–.5*.4=–.4*.5). As shown in Chambers (1991), the power of polarity scores will be reduced by lower correlations between the IV and the DV and by correlations between the IVs.

Uniformity Not Sufficient for Causation

With so much talk about uniform distributions, there is temptation to think that all uniformly distributed variables will polarize as causes. This is not true. Uniform sampling is not the source of polarization; the origin is the *combination* of uniform variables. Uniform sampling of uniform causes only improves the power of corresponding correlations. Uniform sampling does not create the causes. This is illustrated by drawing uniform samples from pre-existing triangular effects. Consider the 4*4 causal manifold. The variable y is triangular and ranges from 2 to 8. There are fewer cases of extreme y than of midrange y. A uniform sample of y can be created by using duplicates, drawing an equal number of cases per level of y. Thus there might be four cases for each of the values of y, ranging from 2 to 8, for a total of 28=4*7 cases in the sample. The y variable will thus be uniformly distributed in the sample, although it is triangular in the causal manifold. Does corresponding correlations, therefore, indicate that y is the cause of x1 and x2? No. The polarity score across the ranges of uniform y is –.13, revealing that the causes of y are x1 and x2. Thus, even though we sample y as uniform, it is still determined to be the effect, not the cause.

Corrections Based on Artificial Variables

The attenuating impact of normal distributions can make it difficult in practice to know if the absence of polarization is due to non-uniform combinations or to the absence of causation. A test of the adequacy of a measured distribution would be very helpful. Since there are a near infinite number of intermediate distributions, between the uniform and normal, tables for evaluating each distribution would need be very large. A simpler way to evaluate a measured distribution is by creating artificial composites. Assume we wish to determine if variables A and B are causes of C but find no significant polarization. We can test to see if this is a distributional problem by simply adding the z-scores of A and B to form C^{art}. Since C^{art} is a perfect composite of A and B, the polarization score for this data should define the upper limit or ceiling for valid inference, making possible a correction based on the potential polarization for the given data. This same correction should be useful when the correlations between the variables deviate from their maximally efficient values (r=0 between IVs and r=.71 between DVs).

Further research on artificial variables is necessary. It is possible that even with these corrections, some distributions will need to be filled out to uniformity by collecting more data. The C^{art} test should at least give some indication of which variables need to be better sampled.

Spurious Correlations and Confounded Variables

Variables are easily confounded in the haphazard flow of events. Measures often clump different phenomena together, with wholes being mislabeled as parts and parts as wholes. Spurious correlations are an example of such confounding. Two variables may correlate with one another but have no direct causal relationship. They may, for example, both be dependent on a common cause, which forms the basis of their correlation. This can be simulated by the model y1=x1+x2 and y2=x1+x3, in which x1 is the common cause of both y1 and y2. Note, however, that y1 could be confused with either x1 or x2, because it includes both. Similarly, y2 could be confused with either x1 or x3, since it contains both. Thus although y1 and y2 are correlated dependent variables with no causal relationship between them as wholes, part of y1 (x1) does cause y2 and part of y2 (x1) does cause y1. This kind of complexity is very difficult to track using structural equations modeling.

Table 3 contains average polarization scores for each of the variables in the y1 and y2 model, based on n=100 with 200 replications of each model.

Table 3

Squared Correlations for Spurious Relations

Sorting by y1 Extremes of y1						Mid-range of y1					
	x1	x2	x3	y1	y2		x1	x2	x3	y1	y2
x1	1					x1	1				
x2	.31	1				x2	–.65	1			
x3	–.00	–.00	1			x3	.00	–.00	1		
y1	.77	.78	.00	1		y1	.12	.09	–.00	1	
y2	.54	.18	.46	.42	1	y2	.46	–.30	.55	.06	1
Sorting by y2 Extremes of y2						**Mid-range of y2**					
	x1	x2	x3	y1	y2		x1	x2	x3	y1	y2
x1	1					x1	1				
x2	.00	1				x2	.00	1			
x3	.33	.00	1			x3	–.64	–.00	1		
y1	.54	.45	.19	1		y1	.46	.56	–.30	1	
y2	.78	.00	.78	.43	1	y2	.11	–.00	.11	.05	1

The squared correlation (retaining sign) between x1 and x2 in the extremes of y1 is r^2=.31. In the midrange of y1 it is r^2=–.64, producing the polarization score of –.20=.31*–.64. In the y2 model the polarization score is –.21=.33*–.64. These results are consistent with previous findings. The variables x1 and x2 cause y1 while x1 and x3 cause y2. The table also shows, however, that in the y1 model polarization occurs with the correlations between y2 and x2; r^2=.18, r^2=–.30, polarization = –.054. A similar configuration occurs for the y2 model, in which y1 and x3 correlated r^2=.19 and r^2=–.30, producing a polarity of –.057. These latter polarization scores demonstrate the effect of confounded variables.

Algebra can be used to untangle the confounding in models y1 and y2. Starting with model y1, Table 3 suggests:

y1=x1+x2	polarity= –.20= .31 * –.65
y1=x2+y2: y1=x2+(x1+x3)	polarity= –.05= .18 * –.30
y1 does not equal x1+x3	polarity= -.00= -.00 * .00

Therefore: y1=x2+x1, not y1=x2+y2.

Since x1 and x3 have zero polarity for y1, x3 cancels out. It is the x1 in y2 that appears to be causal, not the whole of y2. In a similar way, Table 3 suggests the following for model y2:

y2=x1+x3	polarity= –.21= .33* –.64
y2=x3+y1: y2=x3+(x1+x2)	polairty= –.05= .19 * –.30
y2 does not equal x1+x2	polarity= .00= .00 * .00

Therefore: y2=x3+x1, not y2=x3+y1.

Thus the x2 cancels out in y2, illustrating that x1 is the true cause of y2 and not the whole of y1.[6]

Algebraic expressions, such as those above, could be used with indefinitely large data sets. The result would be a parsimonious delineation of inclusive relationships that exist between several and diverse parts and wholes. To experimenters, such data pose a serious challenge to programmatic research but one that can be solved so long as the variables can be physically manipulated and time is unlimited. For those who can not manipulate their variables and know the press of time, corresponding correlations offers hope.

[6]In corresponding correlations (Chambers, 1991) this canceling process was done in the calculation D=rde(y1)–rde(y2). The confound is circumvented because the rde values cancel one another to zero.

Universal Laws, Causes, and Reverse Causes

The most common objection to corresponding regressions has been the belief that all functions are reversible. This argument suggests that because (a=b+c) and (c=a–b) are logically consistent with one another as equations, both are equally compelling as causal models. This argument often assumes that we restrict analysis to scalars, that is, to single values of a, b and c. In this case, the n for the sample is equal to 1 and the single case represents an entire population, without variation across time or context. Such a sample would describe a universe or population, in which both a=b+c and c=a–b are tautological and contradictory! This might seem to hopelessly confound the search for single causes but the situation is not that desperate. The tautology actually defines a law, not a cause. Causes would require that the equations define separate potential instantiations, across independent cases. The same law, however, exists across different causal generations.

Consider Boyle's law of gases, in which the regular proportions of pressure, temperature and volume are defined for a gas:

Volume = temperature/pressure
Temperature = volume*pressure
Pressure = temperature/volume

Boyle's law does not define a cause but, instead, defines the potential forms of various causes. Each equation above could be an independent causal function. Consider three causal models, each conforming to Boyle's law. In the first experiment there are 100 cases, with each being a collection of gases having the same initial volume. The temperature and pressure of these gases are manipulated according to manifold or uniform random conjugation. The result will be that these alterations of temperature (IV1) and pressure (IV2) will cause different levels of volume (DV). In the second experiment the temperature (DV) is caused by conjugating different volumes (IV1) and pressures (IV2). In the third experiment, the pressure (DV) is determined by varying levels of temperature (IV1) and volume (IV2). Thus, across the three experiments, the variables play different causal roles. Within each experimental condition, a different causal model is developed. The n cases are the material, that is defined by the particular lawful equation, that together constitute the hylemorphic generation.

The variety of causes developed across the three experiments is made possible by the use of separate vectors of values instead of simultaneous scalars. The values of the independent variables vary across the 100 trials per experiment. It is from these vectors that we can articulate the causes as consistent with the general law while having their separate histories or profiles. If a researcher samples a simultaneous mixture of all the separate causes, however, corresponding correlations will indicate no cause at all. The different causes will be confounded and cancel one another. It would be possible, however, to examine those cases that failed to fit the polarization pattern while testing each variable as the hypothetical effect. Those cases that fit one model (a=b+c) would not fit the alternative (b=a–c). Cases that fit one model but not the other could be partitioned into separate groups. The researcher could then examine the separate groups for characteristics that would further distinguish the groups.

The above strategy might be called a mediation analysis, in which causal interactions are investigated either by hypothesis or by exploratory strategies. If the data from the three experiments described above were thrown together, we might find that the errors divide the cases into three groups. Further inquiry might show that the data for each group were collected by different experimenters. This would lead the researcher to further inquire as to differences in the actions of the experimenters. The same search strategy would work when dealing with non-experimental data. Instead of finding different experimenters, however, we might find different genders, ages, races, disease status, and so forth for the groups. The mathematics of such exploratory corresponding correlations would not be difficult but will not be developed in this paper.

A final possibility of relevance to this section is *reverse causality*. This is when two related root causal models (y1=x1+x2) and (y2=x1–x2) combine to form a reverse cause (x1=(y1+y2)/2). y1 and y2 are root effects generated from the uniform generations of x1 and x2. y1 and y2 will be uncorrelated with one another and triangularly distributed. Adding y1 to y2, however, recovers the original x1, thus reversing the root causation. This would seem to be a paradoxical case that can not be resolved by corresponding correlations. There will be, in fact, a polarization suggesting that y1 and y2 (effects) cause x1 (their cause)! But the degree of the reverse causation polarization, based on x1=y1+y2, will be less, on average, than it is for the root cause. The average root polarization for 100 replications of sample size 100 was –.51, std=.06. The average polarization for the reverse cause model was –.23, std=.06. The difference between these means is highly significant, t=30.32, p.<.001, df=398. The greater polarization of the root model is probably due to the fact that the reverse cause is based on causes that are disjunctive, thus attenuating the negative correlation in the extremes of x1. These results demonstrate that the polarization measure reflects both the root and reverse causes while allowing us to distinguish between the two types of causes.

Axiomatics and Psychological Measures

The arguments in this paper are largely based on mathematical axioms that define ratio or interval scales. For example, the arguments concerning disjunctive and conjunctive causes derive from the axioms of linear interval sequences. Michell (1997) has argued that because many psychological scales lack ratio and interval properties, many statistical methods are used inappropriately in psychology. The problem arises because so little attention is given to creating measures that conserve basic properties of numbers. This might suggest that corresponding correlations and regressions expect too much from psychological data. Are these methods robust to the degradation of nature's ratio and interval constructions to ordinal measures?

To test the robustness of corresponding correlations, the basic y=x1+x2 model was generated, using 100 observations per 200 replications. After generating the model, however, the values of x1, x2 and y were transformed to ordinal ranks. The data revealed that the polarization effect remained strong. The ranks of x1 and x2 correlated r^2=.33 on average in the extremes of y and r^2=–.65 in the midrange of y. The average polarity score of –.21 is comparable to that found without the degradation to ranks. Thus, so long as the data were generated from interval data and only later degraded, corresponding correlations worked.

Corresponding correlations should not work, however, if the causal model is somehow generated from scales that are less than interval. Such phenomena should not be modeled using arithmetic operations. Within the field of psychology, such irrational processes probably do exist but the error is in assigning them numbers at all. The numbers in some data may be so deficient in basic numerical properties that it stretches the imagination to call them numbers. Corresponding correlations may be of some use in distinguishing such attributions from true numbers. Further research along these and Michell's axiomatic lines would probably be useful.

Corresponding Correlations and Structural Equations Analysis

Corresponding correlations/regressions contributes to structural equation analysis by providing criteria for testing hypothesized causes. As the structural equation discipline stands now, hypotheses are developed from theory or common sense. With corresponding correlations, theory will still be important at the stage of selecting relevant variables but the mathematics will inductively draw the causal arrows in the path diagrams. This will free us from dependence on experimentation. Of course, corresponding correlations could be used to test traditional deductive models of structural equations, as well. Deductive models may be less subject to cross-validation problems than inductive models, but in either case, cross-validation should be used.

Conclusions

This paper has been an introduction to corresponding correlations. A review of philosophies of causation suggests that there has long been a need for non-experimental methods of causal inference. The absence of such methods has shaped the development of science, making it more concerned with concrete objects and less tolerant of abstract variables. The experimentalists looked to manipulation as the sensible alternative to intractable abstractions. Astronomers and other researchers, however, could not utilize experiments and were left dependent on intellectual insight. After failing to develop mathematical methods for causal inference, Gauss and others turned to refinements in measurement theory, leading many modern researchers away from the uniformity of conjunctive causes and toward the non-linearities of disjunctive causes. A major thesis of this paper is that these philosophical developments and derivative mathematical and scientific practices delayed the inference of causation from correlations. These philosophies still impeded the consideration, testing and acceptance of corresponding regressions and corresponding correlations.

The second part of this paper addressed technical steps in the calculation of corresponding correlations. It was shown that we may infer causal relations from correlations, including models with reverse causes. An algebraic method was presented that untangles confounded variables and that could be useful in directing programs of research. The advantages of corresponding correlations over structural equation methods, such as LISREL and path analysis, were argued. Corresponding correlations supports causal inference, whereas, LISREL fails to resolve the ambiguity of correlations and is thus of little use in inferring causation. Corresponding correlations could supply important information each time correlations, regression analysis, factor analysis or structural equations are used. Corresponding correlations should someday precipitate major theoretical, clinical and educational changes, justifying further development of the method.

Future research might develop significance tables from the underlying calculus of corresponding correlations. This should also improve the tractability and efficiency of the method. For example, in recent simulations the author found that the more narrowly the ranges are defined the greater the polarization. That is, taking only the most extreme and most moderate values when defining mid versus extreme ranges of the dependent variable, systematically increased the degree of polarization. There was a loss of sample size, however, in this approach. Future research might employ calculus and linear programming to discover an optimal balance of sample size and width of ranges.

Perhaps the greatest impediment to the further development of corresponding correlations and regressions is the long held belief that it is impossible to infer causation from correlations. Hopefully the current article will bring us a little closer to seeing the possibility of causal inference from correlations and the advantages that such inference should offer society.

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