The Journal of Mind and Behavior
Winter 1999, Volume 20, Number 1
Pages 43–56
ISSN 0271–0137

Goedel's Theorem and Models of the Brain: Possible Hemispheric Basis for Kant's Psychological Ideas

Uri Fidelman

Technion, Israel Institute of Technology

Penrose proved that a computational or formalizable theory of the brain's cognitive functioning is impossible, but suggested that a physical non-computational and non-formalizable one may be viable. Arguments as to why Penrose's program is unrealizable are presented. The main argument is that a non-formalizable theory should be verbal. However, verbal paradoxes based on Cantor's diagonal processes show the impossibility of a consistent verbal theory of the brain comprising its arithmetical cognition. It is suggested that comprehensive theories of the human brain and of physical experience are Kantian rather than Platonic ideas. This suggestion is likewise based on arguments related to diagonal processes.

Penrose (1989, 1990, 1993, 1994) proved that Goedel's theorem implies that no formalizable theory of the brain's cognitive function is possible. This restriction of Penrose's proof to formalizable theories follows from the proof of Goedel's theorem. Each symbol of a formalized theory, which includes the natural numbers, is represented by a number, called a Goedel number. A lexicographical order is defined between the symbols, formulas and proofs, and numerical codes (Goedel numbers) are assigned to the formulas and proofs. Thus this theory is represented in arithmetic by these numerical codes. Then a proposition G, with a Goedel number g, stating "there is no Goedel number of a proof of the proposition having a Godel number g," is formulated. There is no proof of G, since it states that it has none. There is also no proof of the negation of G, since this implies that there is a proof of G. A corollary of this theorem is that this theory, which includes arithmetic, includes no proof of its own consistency, since the consistency of this theory

Requests for reprints should be sent to Dr. Uri Fidelman, Department of General Studies, Technion, Israel Institute of Technology, Haifa 32000, Israel.

implies that there is no proof of G, i.e., G is a theorem. It is impossible that G, for which there is no proof, is a theorem. Thus the corollary is proved by negation.

Every formalizable mathematical, physical or computational model of the brain includes numbers; therefore, any such theory includes arithmetic of the natural numbers, explicitly or implicitly. This model is created by the human brain, hence it is represented by some code in the brain. Let us assume that there is a consistent mathematical, physical or computational model of the brain, which represents the cognitive functions required for constructing this model. Consistency of the model is essential, otherwise it is worthless. The existence of this model implies that its encoded representation in the brain is consistent, otherwise the model itself is inconsistent. Therefore, Goedel's theorem implies that any such model of the brain can apply at most to a level of cognition lower than that required for its construction.

Nevertheless, Penrose's Platonic view (namely, that our mind has a direct understanding of reality) causes him to believe that this difficulty may be avoided by methods which will be described below. This article presents arguments why Penrose's program is unrealizable.

Platonic Ideas and Kantian Ideas

Penrose bases his view on his Platonic ontological belief in the reality of mathematical ideas. Accordingly, this discussion is related to the difference between the Platonic and Kantian (1781) notions of "idea." Platonists define ideas as concepts. They believe that concepts have a real existence, independent of the existence of conscious creatures. Platonic ideas obey the laws of logic.

By contrast, according to Kant's terminology, ideas are not part of the phenomena of physical experience, but are related to them. One kind of relation is that ideas are generalizations of experience. The cosmos as a whole is not a phenomenon perceived by us, but a conceptual generalization of the phenomena. Another kind of relation is that the idea is considered to be the cause of all the phenomena of experience (theological idea). According to Kant the logic of experience is limited to organization of the physical phenomena by our mind, and does not apply to ideas. An attempt at such application may lead to a contradiction, i.e., to a paradox. Actual infinity, which we do not observe in our experience, is an example of a Kantian idea. Kant distinguished between cosmological, theological and psychological ideas. These concepts will be defined below. It should be noted that according to Kant physical phenomena are not things in themselves, but subjective mental interpretations or mental creations. Kantian ideas have a larger degree of subjectivity than physical phenomena, since they are not even subjective physical phenomena. Therefore we will not use the term "idea," but qualify it as "Platonic idea" and "Kantian idea."

Penrose holds a Platonic view regarding both mathematics and the physical world, and presents two arguments in favor of Platonism.

(1) The observation that mathematical concepts are timeless and independent of those who discovered them (1994, p. 413).

(2) The success of physical theories in finding a correspondence between mathematics and physical phenomena (1994, p. 415).

However, these two observations may have alternative explanations:

(1) The universal and timeless nature of mathematical concepts may be due to a genetically determined basic similarity in the functioning of all the human brains which create these concepts.

(2) The correspondence between mathematical theories and physical phenomena may be due to the location of their respective origins in the same neural mechanisms of the human brain. This explanation is in line with Kant's view that physical phenomena are subjective interpretations.

We shall review below Penrose's reasons for believing that Goedel's theorem can be circumnavigated and that a consistent model of the cognitive function of the brain is obtainable. Then some arguments negating Penrose's expectation will be presented. These arguments relate our implied inability to achieve complete understanding of our brain's functioning — to Kant's view that psychological ideas cannot be comprehended by the logic of our understanding. This view is extended to Kantian cosmological and theological ideas. This is done by relating the method by which Goedel's theorem is proved, known as a diagonal process, to a neural process which underlies cognizing Kantian ideas. Kantian ideas, and sometimes also diagonal processes, involve paradoxes.

Diagonal Processes and Paradoxes

A diagonal process in mathematics consists of three stages:

(1) Integration of a set S assumed to be the set of all elements having a certain property P.

(2) Presentation of an additional element having the property P, which is not an element of S. Often this new element is S itself.

(3) Conclusion: the set S is not the set of all the elements having the property P. Thus either this conclusion is proved by negation, or a paradox arises.

The first diagonal theorem is Cantor's proof that the set of all real numbers between 0 and 1, presented as infinite decimal fractions, is larger than the set of all natural numbers. The proof comprises these three stages.

(1) We assume that there is one-to-one mutual correspondence C between the two sets. (This assumption is related to the actual infinite character of these simultaneously presented two sets). The property P is to be a real number between 0 and 1, to which a natural number corresponds by C according to the assumption. The set S is the set of all real numbers corresponding by C to the set of all natural numbers.

(2) A real number d between 0 and 1 is constructed as follows: for each natural n, the n-th digit after the decimal point of d is any digit different from the n-th digit of the real number corresponding by C to n. The real number d cannot correspond by C to any natural number n, since its n-th digit differs from that of the real number corresponding by C to n. That is, d is not an element of the set S.

(3) This proof is true for any correspondence C. Therefore there are more real numbers between 0 and 1 than natural numbers.

It should be noted that while construction of d is a potentially infinite process, the proof as a whole is related to actual infinity, which is a Kantian idea. This proof surprises anyone who encounters it for the first time.

An example of a foundational paradox arrived at via a diagonal process is Russell's paradox. It is related to Frege's axiom of abstraction, which states: for every property P there exists the set S of all elements having the property P. If there is no element which has the property P, then S is the empty set. According to Frege S exists as a Platonic idea.

Now we arrive at Russell's paradox by the three stages of a diagonal process.

(1) The property P is being a set which does not include itself as an element. Formally:

$$P(x) \leftrightarrow \neg [x \in x]$$

The set S is the set of all sets having the property P, that is:

$$x \in S \leftrightarrow \neg [x \in x]$$

The set S exists according to Frege's axiom of abstraction.

(2) The additional element which has the property P is the set S itself. This follows from the observation that each set T, all the elements of which are sets having the property P, must also have this property, otherwise one of its elements (which is this set T itself) does not have this property. The set T is an additional element, not included in T, which has the property P. This consideration applies also to the set S.

(3) The observation that S is an additional element which has the property P contradicts the definition of S as the set of *all* sets having the property P. Thus a paradox arises.

It may be claimed that this last argument is not a paradox, but a proof by negation that Frege's axiom of abstraction is not true, or that there are restrictions on application of the $\in$ relation which designates being an element of a set. However, Frege's logic seems to us to be natural, innate and a priori. Therefore this proof by negation is, in fact, a proof of Kant's view that our a priori logic of understanding experience does not apply to actual infinity, which is a Kantian idea. We may say that Kant predicted the possibility of foundational paradoxes in mathematics.

The proof of Goedel's theorem by a diagonal process is related to Russell's paradox. The above mentioned proposition G is obtained from a predicate G(x) stating: "x is a Goedel number of a predicate P having a Goedel number p, and there is no proof of P(p)." A predicate defines the set of all elements satisfying itself, and the Goedel number of a predicate represents this predicate. Accordingly, the predicate, its Goedel number, and the set are, in a way, equivalent. That is, the domain of G(x) is the set of all sets for each of which there is no proof that it includes itself (i.e., the Goedel number of the predicate defining it) as an element. Let g be the Goedel number of G(x). Then neither G(g) [i.e., G] nor its negation has a proof. There is an obvious similarity between this proof and Russell's paradox.

As stated above, some authors consider Russell's paradox as proof by negation that something is wrong with our conventional logic. However, this "proof by negation" provides us no clue as to *what* is wrong. Some believe that the axiom of abstraction is wrong; others that unrestricted application of the $\in$ relation is wrong, etc. Each such opinion is related to a different foundational approach to mathematics. The opinion presented here is the Kantian view that the logic of experience does not apply to Kantian ideas. However, there is no clear logical or mathematical criterion on preference for one of these approaches. The similarity between Russell's paradox and Goedel's theorem may mean that the reason for the former has implications for our problem — whether it is possible to find a model for the brain's cognitive functioning.

Logic and mathematics apparently cannot provide a unique answer to this problem. Since cognition is related to the brain, it is suggested that we may turn to experimental neuropsychology in our search for a solution that can be tested experimentally. Such a suggested theory, which establishes Kant's view on neuropsychology, is presented in the next section. Unlike the competing theories, it can be tested by experimentation.

Diagonal Processes and the Hemispheric Mechanisms: A Theory

The three stages of diagonal processes described above may be explained by the hemispheric theory of the brain's cognitive functioning. Ben–Dov and

Carmon (1976) suggest a model for the construction of cognitive structures in the brain, based on the analytic–synthetic dichotomy of Levy–Agresti and Sperry (1968). According to this model the brain includes an analytic data-processing mechanism lateralized mainly to the left hemisphere, and a synthetic one lateralized mainly to the right. The left hemisphere processes one datum at a time, and transfers it to the right hemisphere, where the data are integrated into a new whole. This new whole is again treated by the left hemisphere as an individual object, and so on. Thus more and more complex cognitive structures are created.

Now we observe how Russell's paradox arises according to this model of Ben–Dov and Carmon (1976).

(1) The set S of all elements having a certain property P (in this case being a set which does not include itself as an element) is integrated by the right hemisphere. The level of performance of this stage is positively correlated with the efficiency of the right hemispheric "synthetic" mechanism.

(2) This set S is then treated by the left hemisphere as a new element. Since this element is a set having the property P, it cannot be an element of itself, i.e., of S. The level of performance of this stage is positively correlated with the efficiency of the left hemispheric "analytic" mechanism.

(3) The outcome of the previous stage contradicts the definition of S as the set of all elements having the property P. Thus Russell's paradox arises. That is, the right hemispheric structure S is disintegrated as the set of all elements having the property P, and its elements can be treated only analytically by the left hemisphere. In this process of disintegration the left hemisphere may "overcome" the right hemisphere which integrated the set S. The level of performance of this stage, and in fact, of the final understanding that a paradox has arisen, is negatively correlated with the difference between the efficiency levels of the right- and left- hemispheres.

Diagonal processes may involve a cognitive conflict between these two cerebral mechanisms, in that the left one does not accept the finality of the comprehensive integrations of its right counterpart. The product of this comprehensive integration may be the set of all sets or the entire cosmos, and is a universal in the Platonic sense. Yet, the left hemisphere "considers" it to be a new individual item, not necessarily included in the comprehensive whole of the right hemisphere.

This cognitive conflict is, in a way, ontological. The right hemisphere may be considered "Platonic." It "expects" its wholistic structure to include all the elements having the property P, irrespective of whether or not they were constructed as cognitive structures previously. This is the meaning of the word "all" in the phrase "*all* elements having the property P," defining the set S of Russell's paradox. On the other hand, the analytic left hemisphere treats only individual elements. Therefore it is, in a sense, "nominalist." It cannot comprehend the right hemispheric final totality of "*all* elements having the property P," and continues presenting additional new elements having this property.

Thus a diagonal process involves an interhemispheric cognitive conflict in which the left hemisphere "overcomes" the right one, and disintegrates the set S. This lack of cooperation between the hemispheres may imply inability of our logic to handle large infinite sets, as in Russell's paradox. It may also imply proof by negation of the existence of larger and larger infinite numbers, as in Cantor's original diagonal process.

Experimental findings, which I believe provide initial empirical evidence for this suggested relation between the hemispheric mechanisms and diagonal processes, are presented in Fidelman (1987, 1988b, 1990a). The main finding is that individuals' scores on the understanding of diagonal processes are correlated significantly and negatively with the differences between the standardized scores of these individuals on right- and left- hemispheric tests. Similar initial experimental evidence for the relation between left- and right-hemispheric test scores and nominalism and Platonism, respectively, is presented in detail elsewhere (Fidelman, 1989, 1990b). However, the theory presented here is independent of these experiments.

Kantian Cosmological Ideas

In the preceding section it is posited that diagonal processes may induce paradoxes of actual infinity, like that presented by Russell, which may be related to the lack of coordination between the hemispheric mechanisms. These paradoxes are associated with Kant's view that the logic of experience does not apply to ideas of pure reason: attempting to apply logic to Kantian ideas may lead to paradoxes. However, diagonal processes are also applicable to creation of transfinite mathematical structures. Presently physicists and cosmologists speak freely about "universes" or "cosmoses" (e.g., Barrow, 1991, Chapter 5). In this section it is explained how a cognitive process similar to diagonal processes may create distinct, subjective cosmoses, outside our own cosmos. This is a cognitive paradox, similar to the paradoxes obtained by diagonal processes.

According to Kant (1781), a cosmological idea is an extension of the perceived physical phenomena to the entirety of physical phenomena. Through our senses we observe one horizon beyond another full of phenomena, but not all phenomena simultaneously. The entirety of phenomena, the cosmos, is not a phenomenon perceived by us, but a Kantian idea. We may hypothesize that the idea of the entirety of phenomena is integrated from the individual phenomena by the right hemispheric mechanism as a Platonic entity. The latter is in turn transferred to the left hemisphere which, unable to "recognize" it in Platonic terms, "perceives" it as a nominalist individual object (Fidelman, 1988a). This perception must take place within some framework of space and time, external to it, and the cosmos is thus disintegrated as the whole of what there is. Accordingly, more such cosmoses may exist as individual objects within this framework of space and time. This may explain the psychological need of some cosmologists like Linde (1983a, 1983b) to create theories describing several distinct cosmoses. This process is similar cognitively to the diagonal process, and may likewise be related to conflict between the two hemispheric mechanisms.

Kantian Theological Ideas

A Kantian theological idea is an explanation of experience, which is not part of experience. Fidelman (1992) suggested that such ideas are similar cognitively to diagonal processes, and involve a similar interhemispheric conflict. Let us consider the idea of primary reason.

One of Kant's categories of understanding is causality. We have an innate belief that everything is caused. Therefore, each cause has its own reason, and we have a potentially infinite series of reasons, without a beginning. This series is integrated into an actually infinite set (i.e., an infinite set of elements existing simultaneously) of reasons. At this juncture an innate need arises for a reason for this actually infinite set of reasons, though each reason has its own reason. Thus this process of attributing reasons continues. Now we integrate the set S of all these reasons. If we attribute a reason to this set S of all reasons, this contradicts the definition of S as the set of all the reasons — in other words, a paradox.

This paradox is similar to the paradox of all ordinals. An ordinal was defined by von Newman as follows:

(1) The empty set is an ordinal, designated by 0.

(2) For each ordinal n, the set including all its elements and this ordinal as elements, is also an ordinal, the immediate successor of n.

Thus the immediate successor of the ordinal 0 is the ordinal $1=\{0\}$, that of the ordinal 1 is $2 = \{0, \{1\}\}$, etc.

(3) If a set w of ordinals includes, together with each of its elements, all the elements of this element, then this set w is an ordinal. This ordinal w is the immediate successor of the set of all the ordinals which are elements of w.

Thus a successor of an infinite series of ordinals without a last element is obtained. This definition implies the following paradox: let W be the set of all ordinals. Then according to (3) W is an ordinal, the immediate successor of the set of all ordinals W. This contradicts W's being the set of all ordinals. This paradox is similar to the paradox of the primary reason.

The paradox of all ordinals is obtained by a diagonal process. Therefore we may expect that this paradox is related to an interhemispheric conflict (see Fidelman, 1988b), as explained above regarding Russell's paradox. The similarity between the paradox of all the ordinals and the paradox of the primary reason may also connect the paradox of primary reason to interhemispheric conflict.[1]

According to the above discussion a relation with diagonal processes is a common feature of the paradoxes related to Kantian ideas of infinity, cosmological ideas, and theological ideas. The view of Kant (1781) is that ideas exist only in our cognition (see also Fidelman, 1987, 1988a). Therefore, these cognitive paradoxes concern mental creations, and not things as they are in themselves. These paradoxes do not mean that the "totality of all the things as they are in themselves," or the "primary reason of them" do not exist as things in themselves. The paradoxes and their relation to the hemispheres mean merely that the structure of our brain imposes some limitations on the ability of the brain to cope with certain problems, concerning mental creations like the Kantian ideas, and the brain's structure may explain the reasons of these limitations. In the next section I will discuss the possibility of extending the relation of the above discussed Kantian ideas (infinity, cosmological ideas and theological ideas) with diagonal processes to a relation of Kantian psychological ideas with diagonal processes.

Kantian Psychological Ideas

Cognition is not a phenomenon of experience. Relating cognition to the persons in our experience (including ourselves as phenomena) is merely an explanation of behavior. Therefore, according to Kant (1781), cognition is a psychological idea, and our logic of experience does not apply to it. That is, an attempt to apply physical theories of experience to explain cognition, for example, by applying a physical theory to explain the brain's functioning, may lead, according to Kant, to a paradox.

Indeed, the concept "cognition" involves the following paradox. We cognize the phenomena of experience. We also cognize our cognizing the phenomena. Then we cognize our cognizing of our cognizing the phenomena,

---

[1]Another theological paradox related to diagonal processes which is discussed in Fidelman (1992) is the paradox of the almighty. The almighty has all the abilities. However, the almighty cannot have the ability of not being almighty, since the application of this ability contradicts the definition of almighty.

and so on. This leads to a diagonal paradox similar to the paradox of all ordinals and the paradox of primary reason. This simple consideration by itself implies that any theory which explains cognition is a cognizing of our cognizing, and may lead to the paradox.

Penrose (1989, 1990, 1991, 1993, 1994) inferred from Goedel's incompleteness theorem that no computational theory and no formalizable theory of the brain is possible. Goedel's theorem is proved by a diagonal process; therefore, this limitation on theories of the brain is analogous with the relation between diagonal processes and Kantian cosmological and theological ideas, and ideas of infinity. However, Penrose's Platonic view influenced him to believe that this difficulty in achieving a theory of the brain's cognitive function may be avoided by one of the following methods:

(1) Non-formalizable (and therefore untranslatable to Goedel numbers) mathematical theory (see Penrose, 1990, p. 494).

(2) A structure of the physical world which may circumnavigate Goedel's theorem (see Penrose, 1994, pp. 381–383).

However, these two methods suggested by Penrose cannot work.[2]

Method 1: Non-formalizable theory. Let us assume that there is a non-formalizable theory of the totality of the brain's cognitive function. This theory should be formulated lingually, i.e., it should be written or spoken. Natural numbers are represented in the brain. Therefore this theory should represent each natural number verbally.

There is a diagonal process argument which is called the paradox of Richard (1906). Let us order the set of all the English sentences which define real numbers between 0 and 1 (presented as infinite decimal fractions) in a lexicographic order. Designate this ordered set of sentences by R. The set R is infinite. Now we define by an English sentence a real number d between 0 and 1. For each natural number n the n-th digit of d after the decimal point is any digit different from the n-th digit of the number defined by the n-th sentence of R. We prove that this sentence cannot be in the list R exactly as we proved Cantor's diagonal theorem. This contradicts the definition of R as the set of *all* the English sentences which define a number between 0 and 1. This is a proof by negation that there is no verbal non-formalizable theory of the entire cognitive functioning of the brain, which includes both lingual cognition and the cognition about the real numbers.

[2]Penrose is aware of the arguments against these two methods of avoiding Goedel's theorem. However, he believes that mathematical knowledge may originaite outside of the mathematical formalism through Platonic "mathematical insight" (1989, pp. 108–112). His "mathematical insight" is that the consequences of Goedel's theorem regarding the physical world and the brain can be circumnavigated. However, it is not clear to me why the arguments against this circumnavigation are not such "mathematical insight."

There is a similarity between this proof and the proofs of both Cantor's theorem and of Goedel's theorem. The lexicographic ordering of the English sentences defining real numbers has the role of Goedel numbers in Goedel's proof. However, Goedel's theorem concerns natural numbers, while Richard's paradox contradicts only the existence of verbal theories of the brain which include real numbers. The following diagonal paradox of Berry (see Russell, 1906) is a generalization of this contradiction to lingually formulated theories of the brain which include only natural numbers.

Let D be the following definition of the natural number n: "n is the smallest natural number which cannot be defined by less than a thousand letters." Such a number exists, since the number of combinations of less than a thousand letters is finite. There is a set of all the verbal definitions of the number n, all of them comprise at least a thousand letters. However, the definition D of n comprises less than a thousand letters, and it is not included in this set. This leads to a contradiction and a proof by negation that there is no verbal non-formalizable theory of the entire cognitive functioning of the brain, which includes both verbal cognition and the cognition about natural numbers.

Method (2): An unusual structure of the physical world. If the mathematical theory describing this world is non-formalizable, then the above discussion applies to it. If this theory is formalized, then Goedel's theorem applies to it, implying that this theory does not include a proof of the consistency of arithmetic. Of course, this unusual theory of the physical world includes arithmetic. If it includes a theorem that arithmetic is consistent, then the theory itself must be inconsistent.

The example of Penrose (1994, pp. 381–383) of an unusual world is based on the theory of relativity. According to this theory space–time is curved and the cosmos is finite and closed. There is a possibility that the time-line is closed, and the future returns to the past. In this world a Turing computing machine can return to its past output, and determine whether it stopped or not. This is contrary to a diagonal-process theorem which states that the halting problem of a Turing machine is unsolvable. However, this world is contradictory, since one can return to the past and kill one's grandmother, thus preventing one's existence. This is an example of the above statement that a physical world which contradicts Goedel's theorem must be a paradoxical world, described by an inconsistent mathematical theory. That is, in each possible model of the physical world Goedel's theorem is true, and it is thus impossible to construct a physical theory of the brain's cognitive function.

Discussion

We observe that all the Kantian ideas — cosmological, theological, psychological, and ideas of infinity — involve paradoxes or diagonal processes which may be due to the lack of coordination, or a conflict, between the hemispheric mechanisms.[3] These paradoxes and proofs by diagonal processes imply limitations on our ability to construct models of the cosmos and of the primary reason. Here this observation has been extended to limitations on our understanding of human cognition.

The view that we can have only a limited understanding of the brain's cognitive functions may seem pessimistic. However, we may extend a theory of the brain including cognition about arithmetic into a wider formalized theory of the brain which proves the consistency of arithmetic. Now Goedel's theorem for this wider theory implies that this theory does not include a proof of the consistency of itself, and so on. Therefore we can obtain a potentially infinite series of increasing theories of cognition including cognition about arithmetic, each explaining a wider domain. Thus we may practically cover as much of cognition as we like. However, there is no certainty that any one of these theories is consistent.

Any hope for overcoming this limitation on our understanding of the brain depends on our ability to understand the neurological reasons for this limitation. This limitation is related to diagonal processes. Therefore the theory and the experiments relating diagonal processes to the hemispheric mechanisms may be a step in the direction of a larger understanding of the brain's cognitive functioning.

This consideration, regarding the limitation on applying scientific reasoning to psychological ideas, may apply also to other types of Kantian ideas. Penrose (1994) suggested applying comprehensive physical theories in order to understand the brain. I suggest applying neuropsychology to the understanding of the limitations imposed by our brain structure on cosmology and physics.

Finally, this work includes two components. One component is "a priori." It is the philosophical and mathematical proof that it is impossible to perform Penrose's program to construct a comprehensive theory of the brain. This component is valid and independent of the hemispheric theory and of the theory relating the hemispheric mechanisms to the Kantian ideas. The argument based on the paradoxes of Richard and of Berry, which contradicts

[3]All the Kantian ideas involve, after their conceptualization, paradoxes which are related to diagonal processes. However, the process of conceptualization of Kantian ideas may involve paradoxes related to Zeno's paradoxes. These paradoxes, unlike diagonal processes, are related to the inhibition of the left hemisphere by the right one (Fidelman, 1988a).

the possibility of a verbal theory of the brain, is independent of neuropsychology. The second component is "a posteriori." This component is a suggested explanation why the program of Penrose cannot be performed. This explanation depends on the theory of hemispheric differences and on the theory that there is a lack of cooperation between the left- and right-hemispheric mechanisms. However, the hemispheric theory, like every scientific theory, is subject to falsification by experimentation. The innovation of this approach lies with the possibility to submit philosophical theories to judgement by the scientific method.

References

Barrow, J.D. (1991). *Theories of everything: The quest for ultimate explanation*. Oxford: Clarendon Press.

Ben–Dov, G., and Carmon, A. (1976). On time, space and the cerebral hemispheres: A theoretical note. *International Journal of Neuroscience*, *7*, 29–33.

Fidelman, U. (1987). Hemispheric basis for paradoxes and diagonal processes in mathematics. *International Journal of Mathematical Education in Science and Technology*, *18*, 61–66.

Fidelman, U. (1988a). Cerebral basis for transfinite structures and cosmological theories. *Methodology and Science*, *35*, 29–45.

Fidelman, U. (1988b). Ordinals and the hemispheres of the brain. *Cybernetics and Systems*, *19*, 109–122.

Fidelman, U. (1989). The biology of physical knowledge. *Kybernetes*, *18*(1), 48–59.

Fidelman, U. (1990a). Hemispheric competition: Learning and unlearning concepts of infinity. *Cybernetica*, *33*, 59–71.

Fidelman, U. (1990b). The biology of mathematical knowledge. *Kybernetes*, *19*(2), 34–52.

Fidelman, U. (1992). Cerebral asymmetry and theological paradoxes. *Symmetry: Culture and Science*, *3*, 421–432.

Kant, I. (1781). *Kritik der reinen Vernuft*. Riga, Latvia: J.F. Hartknoch.

Levy–Agresti, J., and Sperry, R.W. (1968). Differential perceptual capacities in major and minor hemispheres. *Proceedings of the National Academy of Science*, *61*, 1151.

Linde, A.D. (1983a). Chaotic inflation. *Physics Letters*, *B 129*, 177–181.

Linde, A.D. (1983b). Chaotic inflating universe. *JETP Letters*, *38*, 176–179.

Penrose, R. (1989). *The emperor's new mind*. Oxford: Oxford University Press.

Penrose, R. (1990). Author's response. *Behavioral and Brain Sciences*, *13*, 692–703.

Penrose, R. (1991). Response to Tony Dodd's "Goedel, Penrose, and the possibility of AI." *Artificial Intelligence Review*, *5*, 235.

Penrose R. (1993). An emperor still without mind. *Behavioral and Brain Sciences*, *16*, 616–622.

Penrose, R. (1994). *Shadows of the mind*. Oxford: Oxford University Press.

Richard, J. (1906). Les principes de mathématiques et le problème des ensembles. *Acta Mathematica*, *30*, 295–296.

Russell, B. (1906). On some difficulties in the theory of tansfinite numbers and order types. *Proceedings of the London Mathematical Society*, *4*(2), 29–53.